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Further information concerning the preparation of technical papers is contained in the "Technical Publications Handbook" which can be obtained from the Society.

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Journal of the

STRUCTURAL DIVISION

Proceedings of the American Society of Civil Engineers

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Journal of the
STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

DESIGN OF FOLDED PLATE ROOFS

Howard Simpson,[a] A.M. ASCE
(Proc. Paper 1508)

SYNOPSIS

The procedure for the analysis of a single-span folded plate structure is reviewed and discussed with the aid of an illustrative example. The effects of relative displacements of the longitudinal edges are considered.

INTRODUCTION

The economic and aesthetic advantages of curved thin shell roofs, when employed over large unobstructed floor areas, accrue in considerable extent to folded plate roofs (called also "hipped plate" or "prismatic" structures). Moreover, folded plate roofs are usually easier to form, and can more easily handle large concentrated loads. Openings of considerable size usually can be easily provided, although in such cases a peripheral reinforcing rib may be necessary.

Most folded plate structures can be designed quite readily without the use of differential equations or an awkward number of simultaneous equations.

Interest in and use of this type of roof has increased considerably in this country since the publication here of a paper by Winter and Pei on the design theory. Considerable additions recently have been made to our analytical and experimental knowledge. An ASCE Committee has been studying the subject for several years, and is planning to issue a comprehensive report in the near future. The purpose of this paper is to review and discuss the design assumptions and procedures which may be employed for the analysis of a single-span folded plate structure.

Note: Discussion open until June 1, 1958. A postponement of this closing date can be obtained by writing to the ASCE Manager of Technical Publications. Paper 1508 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. ST 1, January, 1958.

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Definitions

The individual planar elements comprising the structure will be termed "plates."

The "length" of a plate is its dimension between the diaphragms or ribs; its "width" is the perpendicular distance between its two longitudinal edges.

In conformance with the terminology introduced by Winter and Pei,(1) "slab" action will be used to refer to bending of the individual plates out of their planes, and "plate" action to bending in their planes. (The latter has been termed "disk effect" by Craemer(2) and "membrane" action by Ashdown.(3))

Assumptions

The following general assumptions customarily are made in analyzing a folded plate structure:

1. The material is homogeneous, uncracked, and elastic.

2. Longitudinal edge joints are fully monolithic and continuous; there is no relative rotation or translation of two adjoining plates at their common boundary.

3. The principles of superposition hold; that is, the structure may be analyzed separately for the effects of its redundants and various external loadings, and the results added.

4. Individual plates possess negligible torsional resistance. Torsional stresses due to twisting of the plates can be ignored.

5. The function of the supporting diaphragms or bents is to supply the end reactions for the plate action and for the longitudinal slab action. They are assumed incapable of providing restraint against rotation of the ends of the plates in their own planes, but may provide some end restraint for longitudinal slab bending.

The following additional assumptions can be made if each plate is relatively long compared with its width (length-to-width ratio of, say, three or more):

6. Longitudinal strain due to plate action varies linearly across the width of each plate ("plane sections remain plane"). The rate of change of strain with respect to width ordinarily will differ from plate to plate, however, from which it can be inferred that there will be some relative displacement of the joints of a cross-section.

7. Longitudinal slab action can be neglected; that is slab bending carries loads applied to the surface of a plate to the longitudinal edges only, as in a one-way slab system.

It has been common practice in the past to disregard the additional transverse and longitudinal stresses resulting from the resistance of the slabs by transverse bending to the relative displacements of the joints. This has been justified as follows: An isolated plate, at a section remote from the end support is able to rotate (twist) rather easily. Thus, in Figure 1(a), points A and B would move to A' and B' (for all practical cases AA' and BB' can be taken as straight lines perpendicular to the original position AB of the plate).

However, if two plates meet at an angle considerably different from 180 degrees, the displacement of the common joint cannot occur without appreciable deflection of one or both plates in their own planes, as shown in Figure 1(b). Figure 1(c) illustrates the true behavior of a typical plate cross-section; the chord connecting the joints sustains a rotation (ψ) plus a deflection (δ).

Since the deflection (δ) is associated with plate bending, which involves spanning of the plates as deep beams between end diaphragms, the relative displacements of the joints of a cross-section usually will be small.

While the above argument is qualitatively correct, it is safe to neglect the stresses resulting from the plate rotations only in structures of certain proportions; in many structures, these stresses may equal or exceed in magnitude the stresses calculated from other causes. While they probably have only a minor effect on the ultimate strength of an otherwise properly reinforced structure, they nevertheless may result in offensive cracking and deflections.

Method of Analysis

The following design procedure is recommended for a relatively long structure (for which all seven of the assumptions listed in the preceding section apply):

1. Resolve each load applied between joints into two components, one vertical and the other parallel to the plate on which it acts.

2. Assume temporarily that the joints are held against translation, and analyze the cross-section of the structure as a continuous one-way slab. Compute the "holding forces" (reactions) provided by the imaginary supports.

3. Apply to the joints forces equal and opposite to the holding forces, and add any other forces applied directly to the joints. Resolve these loads into components parallel to the contiguous plates to obtain plate loads. Add to these plate loads the parallel components found in Step 1 to obtain the total plate loads.

4. Assuming temporarily that each plate bends independently, compute the moments due to plate action, and the corresponding longitudinal stresses.

5. Continuity demands that the longitudinal stresses in two adjacent plates be identical at their common edge. This condition requires that longitudinal shears exist at the joints. It is a corollary that axial loads will usually be present on the plate cross-section, balancing the resultant total shear forces on the longitudinal edges.

The corrected longitudinal stresses are determined by applying a relaxation procedure to the stresses found in Step 4; the plate deflections (δ) are then computed at mid-span.

6. The Step 5 stresses should be corrected for stresses due to relative joint displacements (plate rotations). These stresses are caused by the rotations of those plates that do not have a free edge. The method of solution is somewhat similar to that used in a multi-storied bent, in that an independent analysis is made for an assumed rotation in each plate. First, a slab-action analysis is made for an arbitrary rotation of the plate at the centerline of its longitudinal span. In the case of a simple-span structure, it is assumed[4] that the rotation angle (and hence the corresponding plate loads) varies along

the longitudinal span as a half-wave of a sine curve. Steps 2, 3, 4, and 5 are then repeated for each analysis.

7. The final deflections are now expressed in terms of the numerical results for the no-rotation (Case I) solution, plus those for the various rotation solutions, each multiplied by an unknown factor k_n.

8. Considering now the geometry of the structure, the results of Step 7 are used to calculate each final plate rotation in terms of the k_n.

9. But it is known that the final rotation of each plate is k_n times the arbitrarily assumed rotation. This gives an equation for each plate rotation, and the resulting set of simultaneous equations can be solved for k_n.

10. The final solution is the sum of the Case I solution, plus each of the rotation solutions multiplied by its respective k_n.

Illustrative Example

The structure shown in Figure 2 will be analyzed under its own dead weight. The structure and loading being symmetrical, only one-half the structure need be considered. Joint D obviously does not rotate in slab action, and has no longitudinal shear.

The results of the slab action analysis are given in Figure 3. Only one cross-section need be analyzed, since the loads are all uniform along the longitudinal span of the structure. The dashed vectors represent the uniformly distributed plate weights. A routine moment distribution is performed, using actual plate dimensions for the calculation of stiffness factors, but the projected lengths for the calculation of fixed end moments and shears. The plate load components of the reactions can be found graphically.

The algebraic sum of the plate load components acting on each plate is used to obtain the plate loads in Figure 4. The simple beam bending moments at mid-span are calculated, together with the corresponding "free-edge" stresses.

Figure 5 shows the effect on longitudinal stresses of a shear force (N) on a longitudinal edge. The vectors with full arrowheads are the longitudinal forces on the cut section required to balance the total shear forces. The expressions for the stresses are obtained from the column stress formula ($P/A \pm Mc/I$). The derivations for the stiffness and carry-over factors to be employed in the stress relaxation procedure are self-explanatory.

The stress relaxation for the correction of the free-edge stresses calculated in Figure 4 is shown in Figure 6. The corrected stresses and the corresponding edge shears (calculated from the corrected stresses by statics) are shown in the lower portion of the figure.

The above solution must now be corrected for the stresses resulting from the rotation of plates BC and CD, illustrated in Figure 7.

The moment distribution shown in Figure 8 is similar to that of Figure 3, except that the former is based on an arbitrarily assumed rotation ψ_{20} of plate BC. The resulting free edge stresses and the stresses after the relaxation procedure are shown in Figure 9. A similar analysis is performed for an arbitrary rotation ψ_{30} of plate CD, in Figures 10 and 11.

Results of the calculations for the plate deflections resulting from each of the conditions of loading are given in Figure 12.

The geometric relationship between plate deflections and rotations must now be established. This can be done graphically, by evaluating the effect on plate rotations of a unit deflection of each plate in turn, or analytically, as shown in Figure 13 for plate BC.

The resulting equations are

$$\psi_2 = \frac{1}{h_2}\left[\frac{\delta_1}{\sin\alpha_{12}} - \delta_2(\cot\alpha_{12} - \cot\alpha_{23}) - \frac{\delta_3}{\sin\alpha_{23}}\right]$$

$$= .113\delta_1 - .0277\delta_2 - .0972\delta_3 \qquad (1)$$

$$\psi_3 = \frac{1}{h_3}\left[\frac{\delta_4 - \delta_2}{\sin\alpha_{23}}\right]$$

$$= .0972(\delta_4 - \delta_2) \qquad (2)$$

Now, by definition

$$\psi_2 = k_2\psi_{20} \qquad (3)$$

$$\psi_3 = k_3\psi_{30} \qquad (4)$$

The numerical values for ψ_{20} and ψ_{30} must be computed. From Figs. 8 and 10,

$$\psi_{20} = \frac{H_2}{I_2E} \times 1 = \frac{11.66}{(1/12)\cdot 1\cdot(1/3)^3 \times 1 \times 144} = 26.2 = \psi_{30},$$

where E is taken as 1 kip per square inch, as before.

Substituting the values of Figure 12 in Eq. (1) and equating to Eq. (3),

$$\psi_2 = 26.2k_2 = .113(-93.4 - 8.32k_2 + 10.0k_3)$$
$$-.0277(+41.3 + 1.22k_2 - 1.19k_3)$$
$$-.0972(-34.4 - 0.53k_2 + 2.72k_3)$$

Collecting terms,

$$-27.1k_2 + .897k_3 = 8.35 \qquad (5)$$

Similarly, from Eqs. (2) and (4),

$$-.067k_2 - 26.3k_3 = .670 \qquad (6)$$

Solving Eqs. (5) and (6),

$$\left.\begin{matrix} k_2 = -.309 \\ k_3 = -.025 \end{matrix}\right] \qquad (7)$$

The final results given in Figure 14 are obtained by adding to the Case I solution k_2 times the Case II solution and k_3 times the Case III solution.

The high tensile stresses shown in Figure 14 could not exist, of course, in an ordinary reinforced concrete structure, but they provide a basis for designing the required reinforcement.(5)

In this example the shear force (T) per unit length of joint can be calculated(1) at any point by multiplying N by V_o/M_o, where M_o refers to the simple-beam bending moment due to the plate loads in the free-edge condition, and V_o to the corresponding shear on the plate cross-section. Since the loading is uniform over the whole span, N will vary parabolically along the span and T linearly (passing through zero at mid-span). It is therefore necessary to determine only one point on each curve to establish the values of N and T at any section.

Information concerning the calculation of shear stresses within the plates can be found in the references listed in the bibliography.(1,2,3,6)

The end stiffeners must be designed to take the reactions due to plate bending, and involve no unusual problems.(1,3,7)

CONCLUSIONS

Sufficient analytical and experimental information is now available to enable engineers to undertake the design of almost any folded plate structure without trepidation. The example worked shows that the design procedure is based on familiar techniques and concepts, and is relatively simple to apply.

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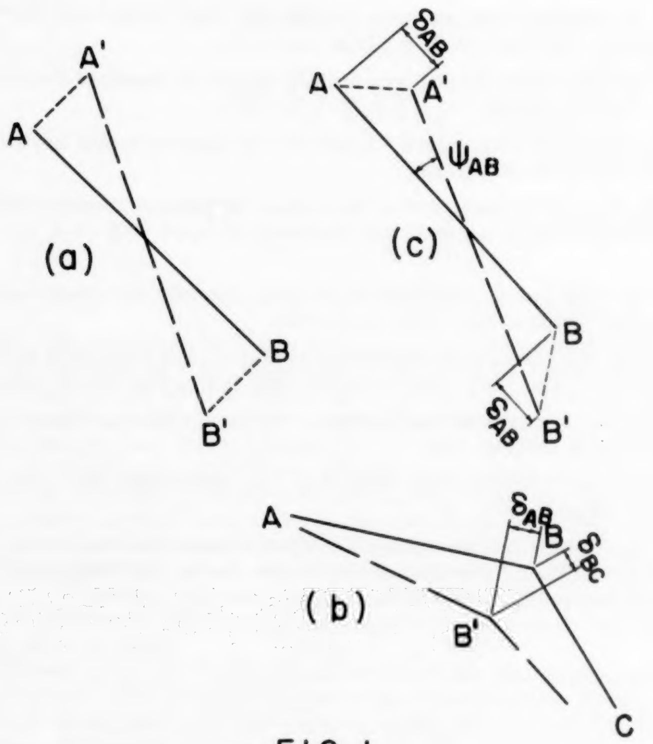


FIG. 1
JOINT DISPLACEMENTS.

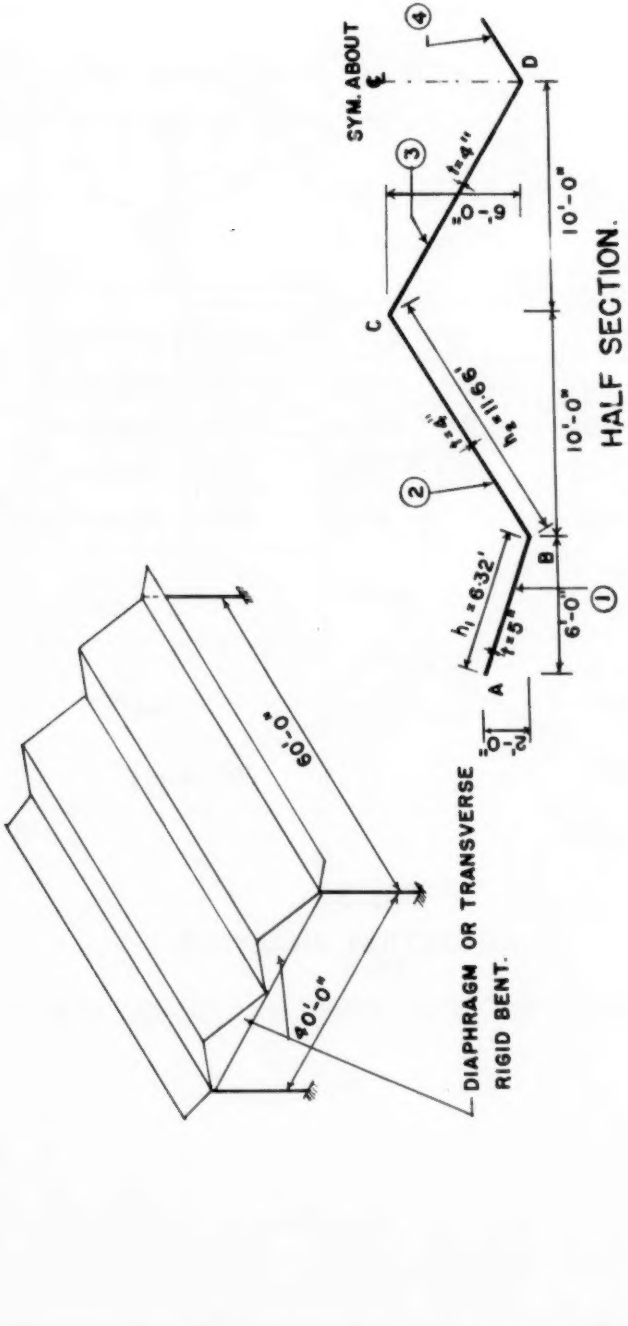


FIG. 2

ILLUSTRATIVE PROBLEM.

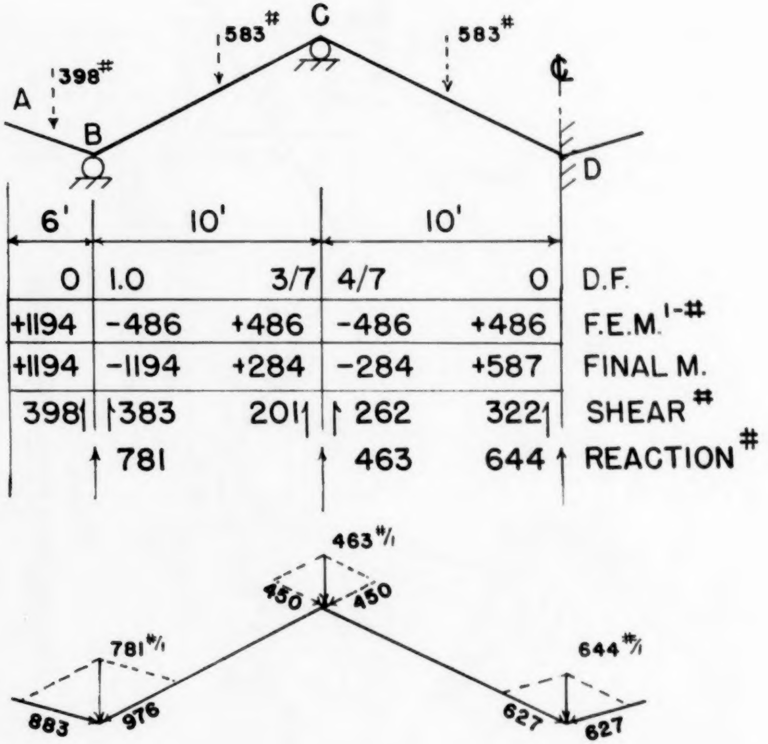


FIG. 3

SLAB ACTION ANALYSIS.

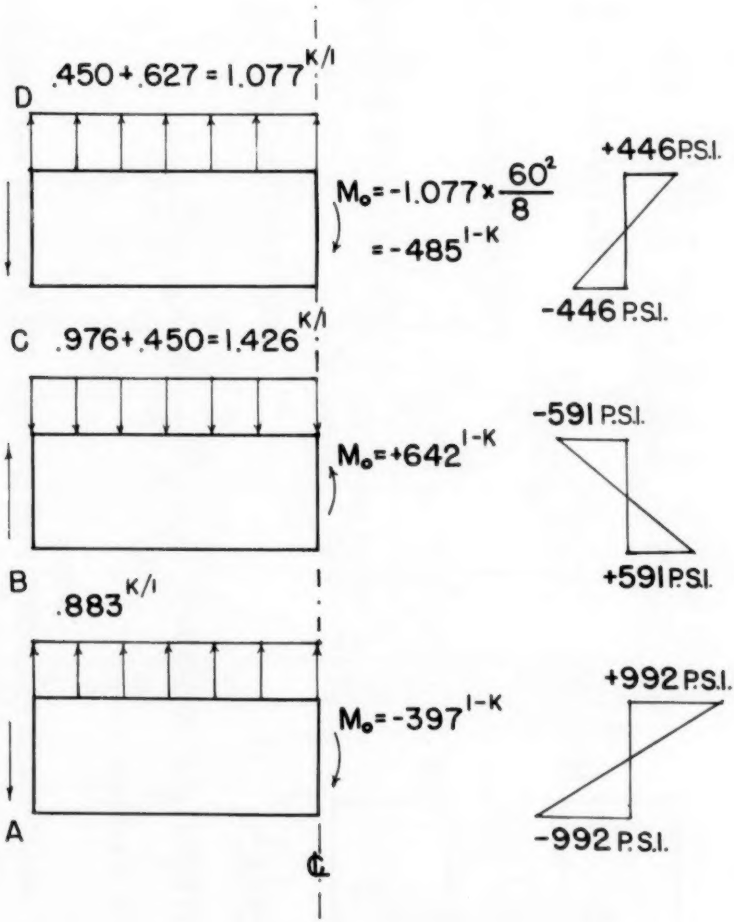
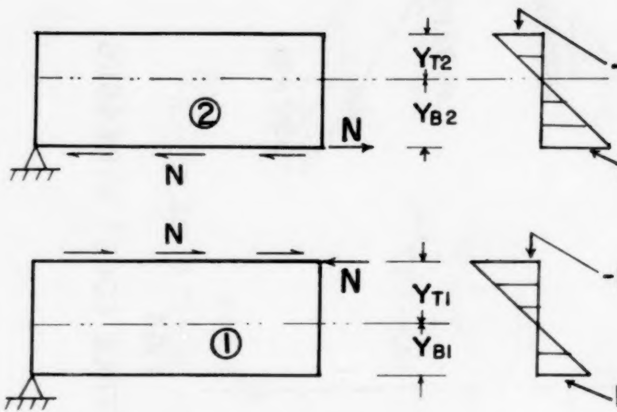


FIG.4

PLATE LOADS AND "FREE EDGE" STRESSES.

$r = \sqrt{I/A}$



$-N \cdot \dfrac{Y_{T2} \cdot Y_{B2} - r_2^2}{I_2}$ FOR RECTANGLE: $= -N \cdot \dfrac{2}{A_2}$

$N \cdot \dfrac{Y_{T2}^2 + r_2^2}{I_2}$ FOR RECTANGLE: $= N \cdot \dfrac{4}{A_2}$

$-N \cdot \dfrac{Y_{T1}^2 + r_1^2}{I_1}$ FOR RECTANGLE: $= -N \cdot \dfrac{4}{A_1}$

$N \cdot \dfrac{Y_{T1} \cdot Y_{B1} - r_1^2}{I_1}$ FOR RECTANGLE: $= N \cdot \dfrac{2}{A_1}$

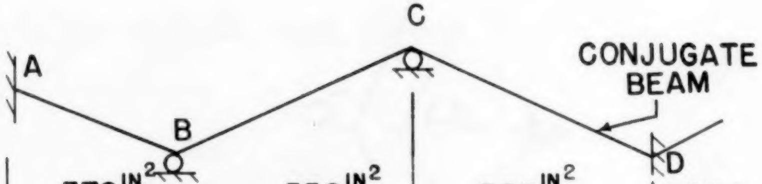
FOR RECTANGLE:

STRESS DISTRIBUTION FACTORS: $K_1 = \dfrac{A_2}{A_1 + A_2}$ & $K_2 = \dfrac{A_1}{A_1 + A_2}$

CARRY-OVER FACTOR $= -\dfrac{1}{2}$

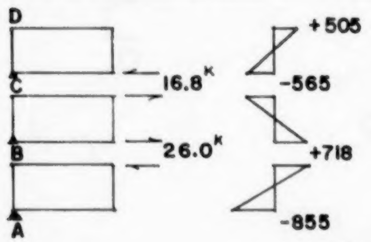
FIG.5

STIFFNESS AND CARRY-OVER FACTORS FOR STRESS RELAXATION PROCEDURE.



379^{IN^2}		559^{IN^2}		559^{IN^2}		AREA
0	.596	.404	.5	.5	0	D.F.
-1/2		-1/2		-1/2		C.O.
-992	+992	+591	-591	-446	+446	F. E.*
	-239	+162	+ 72	- 73		DIST.
+120		- 36	- 81		+ 37	C.O.
	- 21	+15	+41	- 40		DIST.
+10		- 21	- 8		+ 20	C.O.
	- 13	+8	+4	-4		DIST.
+7		-2	-4		+ 2	C.O.
	- 1	+1	+2	-2		DIST.
-855	+718	+718	-565	-565	+505	TOTAL

* FREE EDGE



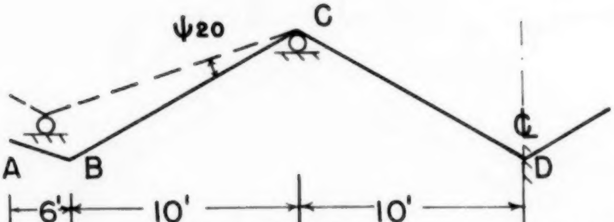
STRESSES ARE IN P.S.I.

FIG.6
CALCULATION OF CORRECTED LONGITUDINAL STRESSES.

FIG.7

PLATE ROTATIONS.

$$F.E.M. = -3E\frac{I_2}{h_2}\psi_{20};\ \text{TAKE } E\frac{I_2}{h_2}\psi_{20} = 1^{1-K}$$



B	C	C	D	
	3/7	4/7		DIST. FACTOR
0	-3.000			F.E.M.
	+1.286	+1.714		DIST.
			+.857	C.O.
	-1.714	+1.714		TOTAL
↑.1714	.1714↓	↓.2571	.2571↑	↑SHEARS
↑.171^K		↓.429^K	.514^K	↑REACTIONS

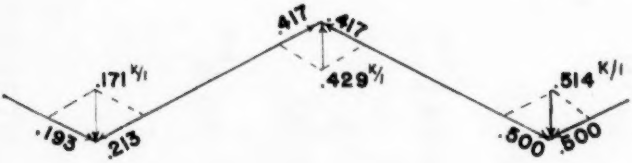


FIG. 8

SLAB ACTION AND PLATE LOADS DUE TO AN ARBITRARY ROTATION OF BC.

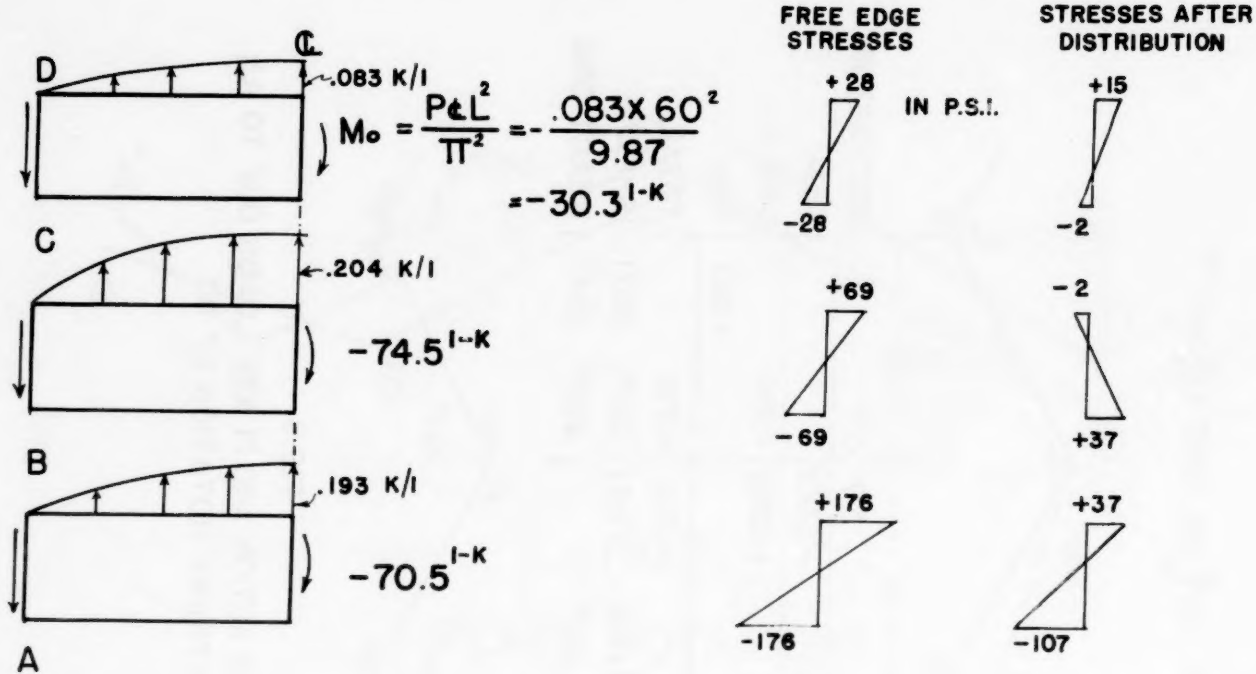
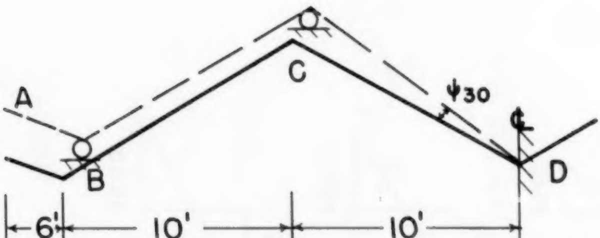


FIG. 9

LONGITUDINAL STRESSES DUE TO BC ROTATION.

$$\text{F.E.M.} = -6E\frac{I_3}{h_3}\psi_{30}\text{; TAKE } E\frac{I_3}{h_3}\psi_{30} = 1^{1\text{-K}}$$



B	C (plate BC)	C (plate CD)	D	
	3/7	4/7		DIST. FACTOR
		-6.00	-6.00	F.E.M.
	+2.57	+3.43		DIST.
			+1.72	C.O.
	+2.57	-2.57	-4.28	TOTAL
.257^K	.257^K	.685^K	.685^K	SHEARS
.257^K		.942^K	1.37^K	REACTIONS

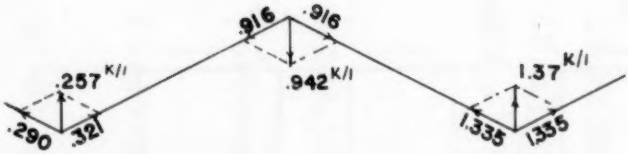


FIG. 10

SLAB ACTION AND PLATE LOADS DUE TO AN ARBITRARY ROTATION OF CD.

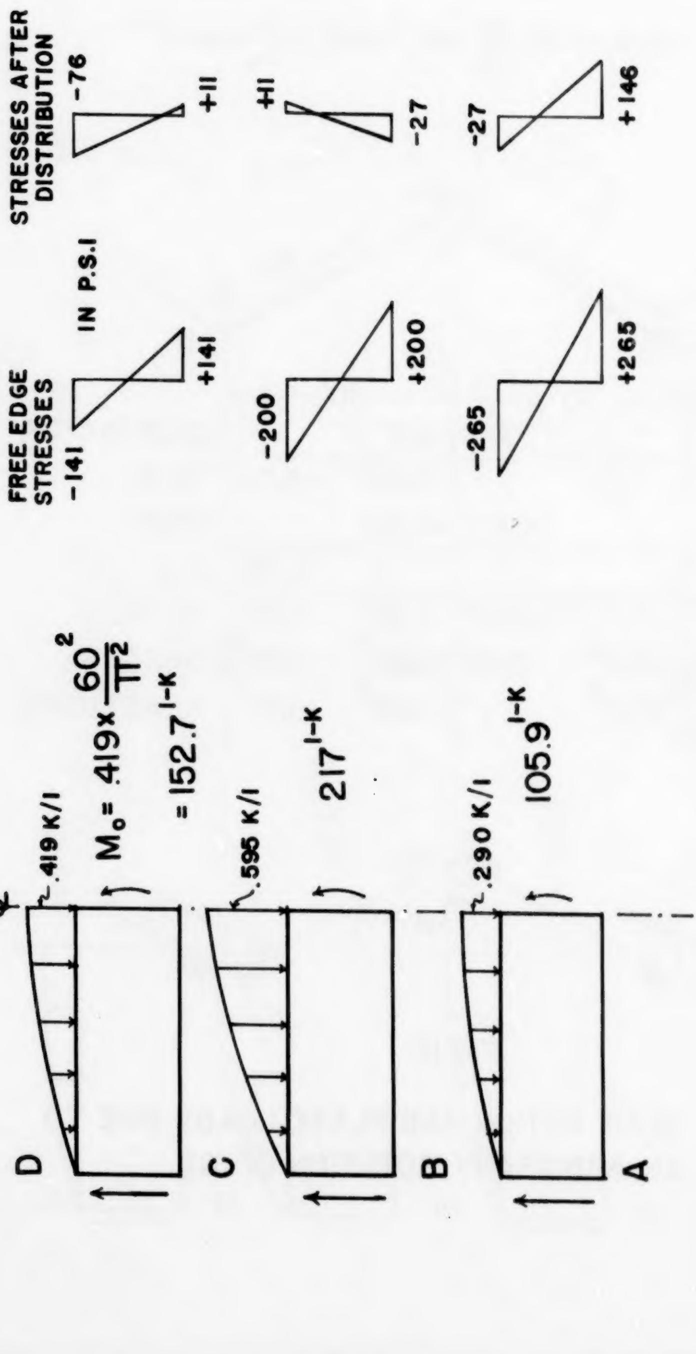
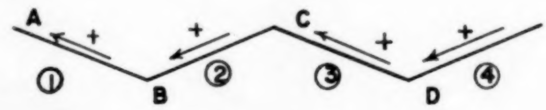


FIG.11

LONGITUDINAL STRESSES DUE TO CD ROTATION.

	FORMULA	δ_1	δ_2	$\delta_3 = -\delta_4$
CASE I ($\psi = 0$)	$\frac{f_B - f_T}{E} \cdot \frac{5L^2}{48h}$	−93.4	+41.3	−34.4
CASE II (ψ_{20})	$\frac{f_B - f_T}{E} \cdot \frac{L^2}{\pi^2 h}$	−8.32	+1.22	−0.53
CASE III (ψ_{30})	$\frac{f_B - f_T}{E} \cdot \frac{L^2}{\pi^2 h}$	+10.0	−1.19	+2.72

SIGN CONVENTION:



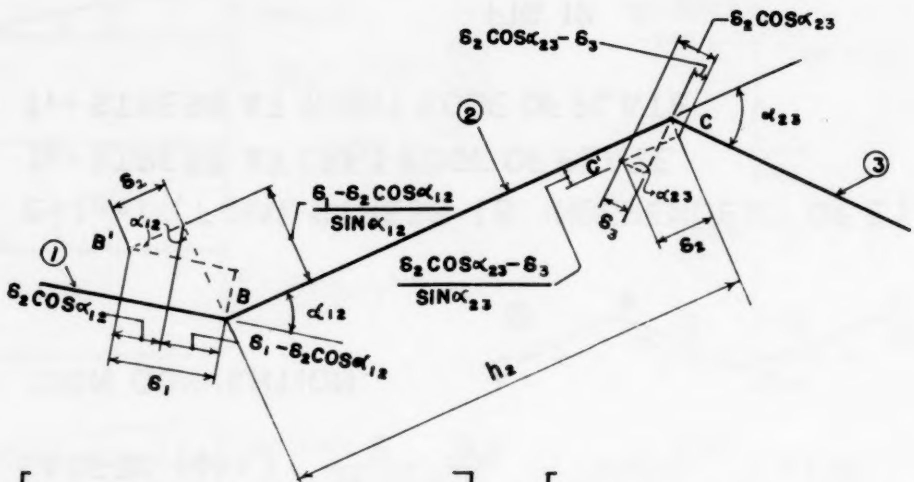
E = 1 K.S.I. (FINAL STRESS IS INDEPENDENT OF E)

f_B = STRESS AT LEFT EDGE OF PLATE

f_T = STRESS AT RIGHT EDGE OF PLATE

FIG. 12

SUMMARY OF DEFLECTION CALCULATIONS.



$$\psi_2 = \frac{1}{h_2}\left[\frac{\delta_1 - \delta_2 \cos\alpha_{12}}{\sin\alpha_{12}} + \frac{\delta_2 \cos\alpha_{23} - \delta_3}{\sin\alpha_{23}}\right] = \frac{1}{h_2}\left[\frac{\delta_1}{\sin\alpha_{12}} - \delta_2(\cot\alpha_{12} - \cot\alpha_{23}) - \frac{\delta_3}{\sin\alpha_{23}}\right]$$

FIG. 13

DERIVATION OF FORMULA FOR ROTATION OF BC IN TERMS OF PLATE DEFLECTIONS.

JOINT	TRANS. MOM. $\psi=0$ 1-#	CORRECTION	FINAL 1-#
B	1194	0	1194
C	284	+467	751
D	587	-158	429

JOINT	LONG. STRESS $\psi=0$ P.S.I.	CORRECTION	FINAL P.S.I.
A	- 855	+ 29	- 826
B	+ 718	- 10	+ 708
C	- 565	+ 1	- 564
D	+ 505	- 4	+ 501

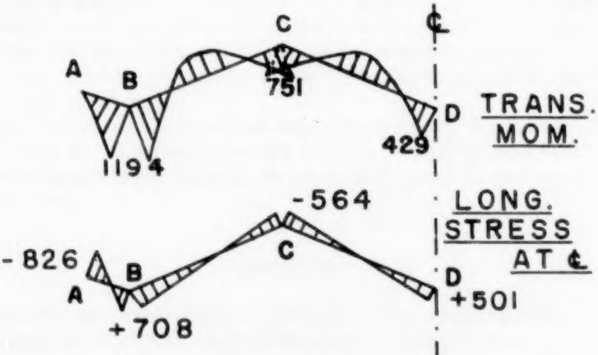


FIG.14

FINAL RESULTS.

Paper 1509 January, 1958 ST 1

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Proceedings of the American Society of Civil Engineers

A METHOD OF DESIGN OF REINFORCED CONCRETE SECTIONS

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SYNOPSIS

In the present paper a general solution is given of the calculation of any section of reinforced concrete with a symmetry axis, eccentrically loaded by the use of computation formulas of the rectangular section and coefficients defined in terms of the statical moment and moment of Inertia of the compressed area of concrete.

The suggested method in certain types of sections is simple and fast.

INTRODUCTION

The purpose of this article is to present a general procedure for the design of reinforced concrete sections as well as for checking purposes. In order to show that the formulas, which are derived herein, are applicable in this general way, the section of Fig. 1 will be used, to illustrate the application of the design procedures.

We assume this section to have an axis of summetry and to be subject to an eccentric load N. The quantities As and A's of reinforcing steel are to be determined by means of the load N, the stresses f_c and f_s and the geometrical data of the section.

Assumptions

1. A plane section before bending remains plane after bending.
2. The variation of stress on a section is rectilinear.
3. The tensile as well as the compressive reinforcement are arranged symmetrically with respect to the section axis of symmetry and evenly distributed alongside the curve (Fig. 1).

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Definition of Symbols

ΔA_s : Quantity per length unit, of tensile reinforcement.

$\Delta A'_s$: Quantity per length unit, of compressive reinforcement.

A_s : Total quantity of tensile reinforcement.

A'_s : Total quantity of compressive reinforcement.

$\overline{f_c}$: Stress of concrete.

f_s : Allowable tensile stress of the steel lower end.

f'_s : Compressive stress of the steel upper end.

K, K' : Centers of gravity of tensile and compressive reinforcement respectively.

e_o, e'_o : Distances of K and K' from the lower end of tensile reinforcement and from the upper end of compressive reinforcement respectively.

t : Depth of section.

d : Depth of section to the lower end of tensile reinforcement.

d' : Distance of the upper (or lower) end of compressive (or tensile) reinforcement from the upper (or lower) fiber of concrete under compression (or tension).

κd : Distance of neutral axis from the upper fiber of compressed concrete.

A_c : Area of the section.

y_κ, y'_κ : Distances of K and K' from the section neutral axis respectively.

$n = \dfrac{E_s}{E_c}$

Calculation of Forces T and T'

From the strain curve (Fig. 2b) we obtain the formulas (1) and (2), which are given in many textbooks:[1,2]

$$f'_s = f_c\, n \left(1 - \frac{1}{\kappa} \, d'/d\right) \tag{1}$$

or

$$f'_s = f_s \, \frac{\kappa - d'/d}{1-\kappa} \tag{1'}$$

The stress of the tensile reinforcement, at a distance y from the neutral axis, is determined as follows: (Fig. 2b).

$$\frac{\varepsilon_{s1}}{\varepsilon_s} = \frac{y}{d(1-\kappa)}$$ and

$$f_{s1} = f_s \frac{y}{d(1-\kappa)} \tag{2}$$

The tensile force T, for a length ds of the curve and at a distance y from the neutral axis is:

$$\Delta T = \Delta A_s \cdot f_{s1} \cdot d_s \tag{3'}$$

Inserting the value of stress f_{s1} in equation (3') we get:

$$\Delta T = \Delta A_s \frac{y}{d(1-\kappa)} \cdot f_s \, d_s \tag{3}$$

The total force T is calculated by integration of the equation (3), the curve of tensile reinforcement being the range of integration, as follows:

$$T = \int \Delta T = \Delta A_s \cdot \frac{f_s}{d(1-\kappa)} \int y \, ds$$ or

$$T = s \Delta As \cdot \frac{f_s}{d(1-\kappa)} \cdot \frac{\int y \, ds}{s}$$ and

$$T = A_s \frac{f_s}{d(1-\kappa)} y_\kappa \tag{4}$$

where: s represents the length of the curve of tensile reinforcement.

Considering that the whole of the tensile reinforcements concentrated at the point K, we obtain from the strain curve (Fig. 2b):

$$\frac{\varepsilon_s^o}{\varepsilon_s} = \frac{y_\kappa}{d(1-\kappa)} = f_s^o / f_s$$ or

$$\frac{f_s^o}{f_s} = \frac{d(1-\kappa) - e_o}{d(1-\kappa)} = 1 - \frac{1}{1-\kappa} \frac{e_o}{d}$$ and

$$f_s^o = f_s \left(1 - \frac{1}{1-\kappa} \frac{e_o}{d}\right) = \rho f_s \tag{5}$$

where: f_s the tensile stress of steel concentrated at the point K

$$\rho = 1 - \frac{1}{1-\kappa} \cdot \frac{e_o}{d} \tag{6}$$

Inserting in equation (4) the ration f_s^o / f_s and the value of f_s^o equation (5):

$$T = \rho A_s f_s \tag{7}$$

Further with the help of strain curve and similar procedure we obtain:

$$T' = \rho' A'_s \, f'_s \tag{8}$$

where:

$$\rho' = 1 - \frac{1}{\kappa} \, \frac{e'_o}{d} \tag{9}$$

In case that the force T' is decreased by a quantity equal to the force deriving from the respective stress of the concrete and the area A' s of the section, the equation (8) gives:

$$T' = \rho' \, \frac{n-1}{n} A'_s \, f'_s \tag{8'}$$

Conditions of Equilibrium

The resultant C of the compression forces is expressed by the equation

$$C = \kappa_1 \, \kappa \, f_c \, A_c \tag{10}$$

where: K_1, is a quantity generally depending on the number K and the geometrical data of the section.
- The distance of the resultant C from the upper end of the section (Fig. 1) is determined:

$$\kappa_2 \, \kappa \cdot d \tag{11}$$

where : K_2 is a quantity generally depending on the number K and the geometrical data of the section.
- The quantities K_1 and K_2 will be determined below:

1) Equilibrium of Forces

$$N = C - T + T' \tag{12}$$

Inserting in (12) the values of C, T and (T!), (10), (7) and (8) respectively:

$$N = \kappa_1 \, \kappa \, f_c \, A_c - \rho A_s \, f_s + \rho' A'_s \, f'_s \tag{13}$$

which divided by A_c and solved for $K_1 \, K \, f_c$:

$$\kappa_1 \, \kappa \, f_c = f_s \left(\bar{p} + \frac{N}{A_c f_s} - \bar{p}' \, \frac{f'_s}{f_s} \right) \tag{14}$$

where:

$$\bar{p} = \rho \cdot \frac{A_s}{A_c} \qquad \bar{p}' = \rho' \, \frac{A'_s}{A_c} \tag{15}$$

Setting:

$$\bar{p}_i = \bar{p} + \frac{N}{Acfs} - \bar{p}' \, {f'_s}/{f_c} \qquad (16)$$

and solving (14) for:

$$\kappa = \frac{1}{\kappa_1} \cdot \frac{f_s}{f_c} \cdot \bar{p}_i \qquad (17)$$

From the stress-distribution diagram the formula (17) is derived:

$$\kappa = \frac{1}{1 + \dfrac{f_s}{nf_c}} \qquad (18)$$

Therefore combining equal expressions (17) and (18):

$$\bar{p}_i = \frac{\kappa_1}{{f_s}/{f_c}\left(1 + {f_s}/{nfc}\right)} \qquad (19)$$

In case the formula (8') is applied, (16) is expressed:

$$\bar{p}_i = \bar{p} + \frac{N}{Acfs} - \bar{p}' \, \frac{(n-1)}{n} \cdot {f'_s}/{f_s} \qquad (19')$$

2) Equilibrium of Moments

Taking the moments with respect to the lower point of the tensile reinforcement gives:

$$Ne = \kappa_1 \kappa f_c A_c d (1 - \kappa_2 \kappa) + \rho' A'_s f'_s (d - \bar{d}') - \rho A_s f_s e_o \qquad (20)$$

where: $\bar{d}' = d' + e'_o$ (21)

Dividing (20) by $A_c d$ and inserting the values of $K_1 K f_c$ and of the number K from (17) gives the equation:

$$\frac{Ne}{A_c d} = f_s p_i \left(1 - \frac{\kappa_2}{\kappa_1} \frac{f_s}{f_c} \bar{p}_i\right) + \bar{p}' f'_s \left(1 - {\bar{d}'}/{d}\right) - \bar{p} f_s \frac{e_o}{d} \qquad (22)$$

Equation (22) is written in the following form:

$$\frac{Mu + \bar{M}u}{Acd} = f_s \bar{p}_i \left(1 - \frac{\kappa_2}{\kappa_1} \frac{f_s}{f_c} \bar{p}_i\right) + \bar{p}' f'_s \left(1 - {\bar{d}'}/{d}\right) \qquad (23)$$

where: Mu = Ne

$$\bar{M}u = \bar{p} f_s e_o A_c = \rho A_s f_s e_o \tag{24}$$

Applying equation (8') makes (23):

$$\frac{Mu + \bar{M}u}{Acd} = f_c \bar{p}_i \left(1 - \frac{\kappa_2}{\kappa_1} \frac{f_s}{f_c} \bar{p}_i\right) + \bar{p}'\left(\frac{n-1}{n}\right) f_s' \left(1 - \bar{d}'/d\right) \tag{23'}$$

Special Case

When reinforcement is arranged in parallel with the neutral axis and at a distance d' from the upper and lower end of the section the quantities ρ, ρ', e_o and e_o' have the values

$$e = e_o' = 0$$

$$\rho = \rho' = 1 \tag{27}$$

Inserting these values in (16), (16'), (23) and (23') gives:

$$\bar{p}_i = \bar{p} + \frac{N}{Acfs} - \bar{p}' \, f_s'/f_s \tag{28}$$

$$p_i = \bar{p} + \frac{N}{Acfs} - \bar{p}' \, \frac{n-1}{n} \, f_s'/f_s \tag{28'}$$

$$\frac{Mu}{Acd} = f_s \bar{p}_i \left(1 - \kappa_2/\kappa_1 \; f_s/f_c \; \bar{p}_i\right) + \bar{p}' f_s' \left(1 - d'/d\right) \tag{29}$$

$$\frac{Mu}{Acd} = f_s \bar{p}_i \left(1 - \kappa_2/\kappa_1 \; f_s/f_c \; \bar{p}_i\right) + \bar{p}' \, \frac{n-1}{n} f_s' \left(1 - d'/d\right) \tag{29'}$$

where:

$$\bar{p} = \frac{As}{Ac} \quad \bar{p}' = \frac{A's}{Ac} \tag{30}$$

Determination of Quantities K_1 and K_2

a) Rectangular section:

From Fig. 4 and formulas (10) and (11) combined, the known values of constants K_1 and K_2 for rectangular section are detained

$$\kappa_1 = \frac{1}{2} \quad \kappa_2 = \frac{1}{3} \tag{31}$$

b) Any Section

Quantity K_1

Take a rectangular section having equal area and depth with the one under consideration (Fig. 5).

$$A_c = b_i \cdot d \tag{32}$$

Since this equivalent rectangular section and the one under consideration have the same depth d, it results from the transformation curve that, for identical stresses f_c and f_s and for reinforcements A_s and A'_s similarly arranged in both sections, both sections have the same neutral axis:

Therefore equation (19) for the equivalent rectangular section is written:

$$p_i = \frac{1}{2\, {}^{f_s}/_{f_c} \left(1 + {}^{f_s}/_{nf_s}\right)} \tag{33}$$

It results from equation (33) above that the quantity p_i of reinforcement of the rectangular section is independent of the arrangement of same in the section.

Dividing the parts of (19) by the parts of (33) respectively gives:

$$K_1 = \frac{1}{2} \; \frac{\bar{p}_i}{p_i} \tag{34}$$

where p_i, the respective quantity of reinforcement of the equivalent rectangular section.

Using (34) determines the quantity K_1 by means of the ration of quantities of reinforcement of the section under consideration and of the equivalent rectangular one K_1 is determined by means of the number K and the geometrical data of the section as follows:

$$f_c' = f_c \; \frac{y}{\kappa d} \tag{35}$$

Force C is given by the equation

$$C = \frac{f_c}{\kappa d} \int y\, dA \tag{36}$$

Equaling (10) and (36) gives:

$$K_1 = \frac{1}{A_c} \; \frac{1}{\kappa^2 d} \int y\, dA \tag{37}$$

where: $\int y dA$ the statical moment of the compressed area of concrete with respect to the neutral axis of the section.

Quantity K_2

The distance y_c of the resultant of the compressive force from the neutral axis is given by the formula:

$$y_c = \frac{\int y^2 dA}{\int y dA} \tag{38}$$

where: $\int y^2 dA$ is the moment of inertia of the compressed area of the concrete with respect to the neutral axis of the section:

Equaling formulas (11) and (38) gives:

$$K_2 = 1 - \frac{1}{\kappa d} \frac{\int y^2 dA}{\int y dA} \tag{39'}$$

Formula (39') is written:

$$K_2 = \frac{1}{3} + u \tag{39}$$

where:

$$u = \frac{2}{3} - \frac{1}{\kappa d} \cdot \frac{\int y^2 dA}{\int y dA} \tag{40}$$

Equations for Rectangular Sections

Inserting in (19) the value of constant $K_1 = 1/2$ gives:

$$p_i = \frac{1}{2 \, f_s/f_c \, (1 + f_s/nf_c)} \tag{41}$$

Equation (41) is the known formula for determination of the quantity of reinforcement for rectangular section with tensile reinforcement under simple bending (3).

a) Equations for rectangular sections with reinforcement arranged in parallel with the neutral axis:

Inserting in equations (29) and (29') the values of constants K_1 and K_2 gives the standard equations of rectangular section:

$$p_i = p + \frac{N}{bd} \frac{1}{f_s} - p' \, f_s'/f_s \tag{42}$$

$$p_i = p + \frac{N}{bd} \frac{1}{f_s} - p' \frac{n-1}{n} \, f_s'/f_s \tag{42'}$$

$$\frac{Mu}{bd^2} = f_s\, p_i \left(1 - \frac{2}{3}\frac{f_s}{f_c} p_i\right) + p' f_s' \left(1 - d'/d\right) \tag{43}$$

$$\frac{Mu}{bd^2} = f_s\, p_i \left(1 - \frac{2}{3}\frac{f_s}{f_c} p_i\right) + p' \frac{n-1}{n} f_s' \left(1 - d'/d\right) \tag{43'}$$

The quantity $f_s p_i \left(1 - \frac{2}{3}\frac{f_s}{f_c} p_i\right)$ issymbolized by K in the tables.(2)

The compressive reinforcement is A' s determined by the known formula:(2)

$$A_s = \frac{Mu - Kbd^2}{d f_s' (1 - d'/d)} \tag{44}$$

$$A_s' = \frac{Mu - Kbd^2}{\frac{n-1}{n} d \cdot f_s' (1 - d'/d)} \tag{44'}$$

The tensile reinforcement As is determined by the equations

$$A_s = p_i\, bd + A_s' \, f_s'/f_c - \frac{N}{f_s} \tag{45}$$

$$A_s = p_i\, bd + A_s' \, \frac{n-1}{n} \cdot f_s'/f_c - \frac{N}{f_s} \tag{45'}$$

b) Equations for rectangular sections with reinforcements not in parallel arranged with the neutral axis.

Equations (16), (16') and (23), (23') in this case have the following form:

$$p_i = p_o + \frac{N}{bd}\frac{1}{f_s} - p_o' \, f_s'/f_s \tag{46}$$

$$p_i = p_o + \frac{N}{bd}\frac{1}{f_s} - p_o' \, \frac{n-1}{n} \, f_s'/f_s \tag{46'}$$

$$\frac{Mu + \bar{M}u}{bd^2} = f_s\, p_i \left(1 - 2/3 \, f_s/f_c \, p_i\right) + p_o' f_s' \left(1 - \bar{d}'/d\right) \tag{47}$$

$$\frac{Mu + \bar{M}u}{bd^2} = f_s\, p_i \left(1 - 2/3 \, f_s/f_c \, p_i\right) + p_o' \frac{n-1}{n} f_s' \left(1 - \bar{d}'/d\right) \tag{47'}$$

where:

$$p_o = \rho \frac{A_s}{bd} \qquad p'_o = \rho' \frac{A'_s}{bd} \tag{48}$$

Equations (44), (44') and (45), (45') have the following form:

$$A'_s = \frac{Mu + \bar{M}u - Kbd^2}{df'_s\,(1 - \bar{d}'/d)} \cdot \frac{1}{\rho'} \tag{49}$$

$$A'_s = \frac{Mu + \bar{M}u - Kbd^2}{\frac{n-1}{n}\,df'_s\,(1 - \bar{d}'/d)} \cdot \frac{1}{\rho'} \tag{49'}$$

$$A_s = \frac{1}{\rho} p_i\, bd + \frac{\rho'}{\rho} \frac{f'_s}{f_s} A'_s - \frac{N}{f_s} \frac{1}{\rho} \tag{50}$$

$$A_s = \frac{1}{\rho} p_i\, bd + \frac{\rho'}{\rho} \frac{n-1}{n} \frac{f'_s}{f_s} A'_s - \frac{N}{f_s} \frac{1}{\rho} \tag{50'}$$

For the case of a rectangular section the moment $\bar{M}u$ has the values:

$$\bar{M}u = \rho A_s f_s e_o \tag{51}$$

Inserting in (25) the value of As (50) gives:

$$\bar{M}u = e_o f_s \left(p_i\, bd + \rho' A'_s \frac{f'_s}{f_s} - \frac{N}{f_s} \right) \tag{52}$$

General Formulas for Design

Inserting in (23) the values of K_1 and K_2, equations (34) and (39) gives:

$$\frac{Mu + \bar{M}u}{Acd} = 2K_1 f_s p_i \left(1 - \frac{2}{3} \frac{f_s}{f_c} p_i\right) + p' f'_s \left(1 - \bar{d}'/d\right) - 4K_1 u f_s \frac{f_s}{f_c} p_i^2 \tag{53}$$

and

$$\frac{1}{2K_1} \frac{Mu + \bar{M}u}{Acd} = f_s p_i \left(1 - \frac{2}{3} \frac{f_s}{f_c} p_i\right) + \frac{\bar{p}'}{2K_1} f'_s \left(1 - \bar{d}'/d\right) - 2u f_s \frac{f_s}{f_c} p_i^2 \tag{54}$$

Substituting the area of the equivalent rectangular section for the area of the one under consideration ($A_c = b_1 d$) gives:

$$\frac{1}{2K_1}\frac{Mu+\bar{M}u}{b_i d^2}+\frac{M\overset{o}{u}}{b_i d^2}=f_s p_i\left(1-\frac{2}{3}\frac{f_s}{f_c}p_i\right)+\frac{\bar{p}'}{2K_1}f_s'\left(1-\bar{d}'/d\right) \tag{55}$$

where:

$$M\overset{o}{u}=\beta u\, b_i d^2 \qquad \bar{M}u=\rho A_s f_s e_o \tag{56}$$

and

$$\beta=2f_s\frac{f_s}{f_c}p_i^2 \tag{57}$$

Inserting in (16) and (16') the value of $\bar{p}_i$ (equation 34) and dividing by $2K_1$ gives:

$$p_i=\frac{\bar{p}}{2K_1}+\frac{N}{b_i d}\quad\frac{1}{2K_1 f_s}-\frac{\bar{p}'}{2K_1}\quad f_s'/f_s \tag{58}$$

$$p_i=\frac{\bar{p}}{2K_1}+\frac{N}{b_i d}\quad\frac{1}{2K_1 f_s}-\frac{\bar{p}'}{2K_1}\quad\frac{n-1}{n}\quad f_s'/f_s \tag{58'}$$

Substituting in (15) the area of the equivalent rectangular section for A_c gives:

$$\bar{p}=\rho\frac{A_s}{b_i d}\qquad \bar{p}'=\rho'\frac{A_s'}{b_i d} \tag{59}$$

Inserting in (55), (58) and (58') the values of $\bar{p}$ and $\bar{p}'$, (equations 59) gives:

$$\frac{M_T}{b_i d^2}=f_s p_i\left(1-\frac{2}{3}\frac{f_s}{f_c}p_i\right)+\frac{\rho'}{2K_1}\frac{A_s'}{b_i d}f_s'\left(1-\bar{d}'/d\right) \tag{60}$$

$$\frac{M_T}{b_i d^2}=f_s p_i\left(1-\frac{2}{3}\frac{f_s}{f_c}p_i\right)+\frac{\rho'}{2K_1}\frac{A_s'}{b_i d}\frac{n-1}{n}f_s'\left(1-\bar{d}'/d\right) \tag{60'}$$

$$p_i=\frac{\rho}{2K_1}\frac{A_s}{b_i d}+\frac{N}{b_i d}\quad\frac{1}{2K_1 f_s}-\frac{\rho'}{2K_1}\frac{A_s}{b_i d}\quad f_s'/f_s \tag{61}$$

$$p_i=\frac{\rho}{2K_1}\frac{A_s}{b_i d}+\frac{N}{b_i d}\quad\frac{1}{2K_1 f_s}-\frac{\rho'}{2K_1}\frac{A_s}{b_i d}\quad\frac{n-1}{n}\quad f_s'/f_s \tag{61'}$$

where:

$$M_T=\frac{Mu+\bar{M}u}{2K_1}+M\overset{o}{u} \tag{62}$$

Inserting in (56) the value of reinforcement A_s (61) gives:

$$\bar{M}u=e_o f_s\left(2K_1 p_i\, b_i d+\rho' A_s\, f_s'/f_s-\frac{N}{f_s}\right) \tag{62'}$$

The moment $\overline{M}u$ is determined in the general case an approximation. With the help of (62'), the $\overline{M}u$ is determined for $A's = o$. Then M_T and reinforcement $A's$ are determined from (65) or (65'). Again $\overline{M}u$ is calculated for $A's$ as above determined and then the procedure for determining $A's$ is repeated.

Consequently the equations for determination of reinforcements As and $A's$ are:

$$A_s' = \frac{M_T - K b_i d^2}{d f_s' (1 - d'/d)} \frac{2\kappa_1}{\rho'} \tag{63}$$

$$A_s = \frac{M_T - K b_i d^2}{\frac{n-1}{n} d f_s' (1 - d'/d)} \frac{2\kappa_1}{\rho'} \tag{63'}$$

$$A_s = \frac{2\kappa_1}{\rho} p_i b_i d + \frac{\rho'}{\rho} \frac{f_s'}{f_s} A_s' - \frac{N}{f_s} \frac{1}{\rho} \tag{64}$$

$$A_s = \frac{2\kappa_1}{\rho} p_i b_i d + \frac{\rho'}{\rho} \frac{f_s'}{f_s} A_s' \frac{n-1}{n} - \frac{N}{f_s} \frac{1}{\rho} \tag{64'}$$

In the case of a not rectangular section where the tensile as well as the compressive reinforcement are arranged in parallel with the neutral axis, the equations (63), (63'), (64) and (64') come under the form:

$$A_s' = \frac{M_T - K b_i d}{d f_s' (1 - d'/d)} \cdot 2\kappa_1 \tag{65}$$

$$A_s' = \frac{M_T - K b_i d}{\frac{n-1}{n} d f_s' (1 - d'/d)} \cdot 2\kappa_1 \tag{65'}$$

$$A_s = 2\kappa_1 p_i b_i d + f_s'/f_s \, A_s' - \frac{N}{f_s} \tag{66}$$

$$A_s = 2\kappa_1 p_i b_i d + f_s'/f_s \, \frac{n-1}{n} A_s' - \frac{N}{f_s} \tag{66'}$$

CONCLUSION

As a basic rule the design of a reinforced concrete section of uncommon form is performed by means of graphical method. We consider this, however, to be neither fast nor handy. Furthermore it contains the handicap of incorrectness, pertaining to all graphical methods.

We have endeavoured in this treatise to present an analytic method and solution of the problem, independently of the form of section. This solution, as it will be obvious from the examples that follow concerning certain forms of section, is reached faster than by graphical method.

When more complicated forms of section are considered, graphical procedure is used only for the determination of the statical moment and the moment of inertia of the compressed area of concrete.

Beside this, the formulas for rectangular section with compressive or tensile reinforcement, arranged in parallel or not with the neutral axis, may be considered as the generalized formulas for the design of rectangular section.

With the help of tables or diagrams for the rectangular section with tensile reinforcement in parallel arranged with the neutral axis, we can proceed to the design of the rectangular section, subject to an eccentric load and with reinforcement not in parallel arranged with the neutral axis of the section.

Various Forms of Section

T - Section

For a T - section (Fig. 7) we can consider that the area Ac is the corresponding to the depth d of the T-section.

1) Determination of bi:

$$b_i = \nu b \tag{67}$$

in which:

$$\nu = 1 - \left(1 - \frac{b_o}{b}\right)\left(1 - \frac{d_o}{d}\right) \tag{68}$$

2) Determination of $2K_1$:

$$\int y\,dA = \frac{b(\kappa d)^2}{2}\left[1 - \left(1 - \frac{b_o}{b}\right)\left(1 - \frac{d_o}{\kappa d}\right)^2\right] \tag{69}$$

Combining equations (37) and (69) gives:

$$2K_1 = \frac{a}{\nu} \tag{70}$$

where: a =

$$1 - \left(1 - \frac{b_o}{b}\right)\left(1 - \frac{d_o}{\kappa d}\right)^2 \tag{71}$$

The coefficient a is determined with the help of table I

3) Determination of u

$$\int y^2\,dA = \frac{b(\kappa d)^3}{3}\left[1 - \left(1 - \frac{b_o}{b}\right)\left(1 - \frac{d_o}{\kappa d}\right)^3\right] \tag{72}$$

Combining equations (40), (69) and (72) gives:

$$u = \frac{2}{3}\left[1 - \frac{1 - \left(1 - b_o/b\right)\left(1 - d_o/\kappa d\right)^3}{1 - \left(1 - b_o/b\right)\left(1 - d_o/\kappa d\right)^2}\right] \tag{73}$$

The coefficient u is determined with the help of table II.

Example

Given a T-section subject to bending moment M = 2.000.000 in. lb. combined with axial load N = 13.000 lb.,

Also given $b_o = 8\ in$, $b = 40\ in$, $d_o = 4\ in$, $d = 25\ in$, $d' = 2\ in$

Allowable stress in concrete $f_c = 700\ lb/sq\text{-}in$

Allowable stress in steel $f_s = 20000\ lb/sq\text{-}in$

$$\varkappa d = 0.344 \times 25 = 8.6\ in > d_o \qquad n = 15$$

$$Mu = 2.000.000 + \frac{25}{2} 13000 = 2.161.000\ in\text{-}lb$$

$$b_o/b = \frac{8}{40} = 0.20 \qquad d_o/d = \frac{4}{25} = 0.160 \qquad d_o/\varkappa d = \frac{4}{8.6} = 0.405$$

$$V = 1 - (1 - 0.20)(1 - 0.160) = 0.388 \qquad b_i = 0.388 \times 40 = 15.5\ in$$

$$K b_i d^2 = 107 \times 15.5 \times 25^2 = 1.035.000\ in\text{-}lb$$

$$\beta = 2 \times 20000 \times \frac{20000}{700} \times 0.006^2 = 41.2\ lb/sq\text{-}in$$

From table I : a = + 0,773
From table II : u = - 0,089

$$M_u^o = -0.089 \times 41.2 \times 15.5 \times 25^2 = 35500\ in\text{-}lb$$

$$2\varkappa_1 = \frac{a}{V} = \frac{0.733}{0.388} = 1.89$$

$$M_T = \frac{Mu}{2\varkappa} + Mu = \frac{2.163.000}{1.89} - 35500 = 1.114.500\ in\text{-}lb$$

$$1 - d'/d = 1 - 2/25 = 0.92 \qquad f_s' = 700 \times 15 \left(1 - \frac{1}{0.344} \times \frac{2}{25}\right) = 8060\ lb/sq\text{-}in$$

$$A_s' = \frac{1.114.500 - 1.035.000}{25 \times 8060 \times 0.92} = 0.43\ sq\text{-}in$$

$$A_s = 1.89 \times 0.006 \times 15.5 \times 25 + \frac{8060}{20000}\ 0.43 - \frac{13000}{20000} = 3.92\ sq\text{-}in$$

Triangular Section (Fig. 8)

$$A_c = b_i d = \tfrac{1}{2}\, bd \qquad b_i = vb \qquad v = \tfrac{1}{2}$$

Determination of K_1
Fig. 8 gives: $b_K = Kb$

$$\int y\,dA = \kappa d \frac{(\kappa d)^2}{6} \tag{74}$$

Combining (37) and (74) gives:

$$2\kappa_1 = \frac{2\kappa}{3} \tag{75}$$

Determination of u:

$$\int y^2\,dA = \frac{\kappa b(\kappa d)^3}{12} \tag{76}$$

Combining (40), (74) and (76) gives:

u = 1/6

Triangular and Rectangular Sections Combined

1st case: $\kappa d < d_0$ (Fig 9a) $b_i = vd$

in which
$$v = 1 - \tfrac{1}{2}\, \tfrac{d_0}{d} \tag{77}$$

$$2\kappa_1 = \frac{a}{v}$$

in which
$$a = \frac{\kappa}{3} \tag{78}$$

and
$$u = \frac{1}{6} \tag{79}$$

2nd case: $\kappa d > d_0$ (Fig. 9b)

$$2\kappa_1 = \frac{a}{v}$$

$$\alpha = \frac{1}{3} \quad \frac{1-(1-{d_o}/{xd})^3}{{d_o}/{xd}} \tag{80}$$

$$u = \frac{2}{3} - \frac{1}{2} \quad \frac{1-(1-{d_o}/{xd})^4}{1-(1-{d_o}/{xd})^3} \tag{81}$$

The coefficients a and u are determined with the help of Tables III and IV.

3rd case (Fig. 9c)

$$2x_1 = \frac{1}{v} \tag{81}$$

$$v = 1 - \frac{1}{2}\,{d_o}/{t} \tag{82}$$

$$u = 0$$

$$e_o = \frac{d_o - d'}{2} \tag{83}$$

Example

For the section of Fig. 9b subject to bending moment M determined As and A' s:
Given: b = 12 in, d = 28 in, do = 8 in, M = 850.000 in.lb.

$$f_c = 900\ {lb}/{sq\text{-}in} \qquad f_s = 20000\ {lb}/{sq\text{-}in} \qquad n = 15$$

$$xd = 0.403 \times 28 = 11.3 > d_o$$

$$v = 1 - {1}/{2}\ \frac{8}{28} = 0.857 \qquad {d_o}/{xd} = {8}/{11.3} = 0.708$$

$$\beta = 2 \times 20000\ \frac{20000}{900} \times 0.091^2 = 73.6$$

$$b_i = 0.857 \times 12 = 10.3\ in$$

Tables III and IV give:

a = 0.465 u = 0.156 $\qquad 2x_1 = \frac{0.465}{0.857} = 0.542$

$$M_T = \frac{850000}{0.542} + 73.6 \times 0.156 \times 10.3 \times 28^2 = 1.663.000$$

$$K b_i d^2 = 157 \times 10.3 \times 28^2 = 1.270.000\ in\text{-}lb$$

The reinforcement As' is arranged in parallel with the sides of the triangle and at a distance d_o from the angle (Fig. 9b)

$$d' = 2\ in \qquad \bar{d}' = d' + e_o' = 2 + \frac{8-2}{2} = 5\ in \qquad e_o' = 3\ in$$

$$f_s' = 900 \times 15\left(1 - \frac{1}{0.403} \cdot \frac{2}{28}\right) = 11.150\ lb/sq\text{-}in$$

$$\rho' = 1 - \frac{1}{0.403}\ \frac{3}{28} = 0.734$$

$$A_s = \frac{1\,663\,000 - 1\,270\,000}{28 \times 11.150\left(1 - 5/28\right)} \cdot \frac{0.542}{0.734} = 1.04\ sq\text{-}in$$

$$A_s = 0.542 \times 0.0091 \times 10.3 \times 28 + \frac{11\,150}{20000} \times 1.04 \times 0.734 = 1.85\ sq\text{-}in$$

Example

For the section of Fig. 9c when given:

b = 10 in **d = 26 in** **do = 12 in** **t = 28 in**

M = 1.000.000 in lb **N = 16.000 lb**

fc = 900 lb/sq-in **fs = 20.000 lb/sq-in** **n = 15**

Determine As and A' s

$$\kappa d = 0.403 \times 26 = 10.5\ in \qquad v = 1 - 1/2\ \frac{12}{28} = 0.786$$

$$b_i = 0.786 \times 10 = 7.86\ in$$

$$2\kappa_1 = \frac{1}{v} = \frac{1}{0.786} = 1.27 \qquad u = 0$$

$$e_o = \frac{12-2}{2} = 5\ in \qquad M_u^o = 0 \qquad \rho' = 1$$

$\overline{M}$u is calculated for A' s = o

$$\overline{M}u = 5 \times 20000\left(1.27 \times 0.0091 \times 7.86 \times 26 - \frac{16000}{20000}\right) = 156000\ in\text{-}lb$$

$$Mu = 1.000.000 \times \frac{26}{2} \times 16000 = 1.208.000\ in\text{-}lb$$

$$M_T = 1.208.000 + 156.000 = 1.364.000\ in\text{-}lb$$

$$f_s' = 15 \times 900 \left(1 - \frac{1}{0.403} \cdot \frac{2}{26}\right) = 10900 \ {}^{lb}/_{sq\text{-}in}$$

$$A_s = \frac{1364.000 - 157 \times 7.86 \times 26^2}{26 \times 10900 (1 - {}^2/_{26})} = 2.04 \ sq\text{-}in$$

Mu is calculated again.

$$\bar{M}u = 156.000 + 2.04 \ \frac{10900}{20000} \times 5 \times 20000 = 267.000$$

$$M_T = 1.208.000 + 267.000 = 1.475.000$$

$$A_s = \frac{1.475.000 - 1.57 \times 7.86 \times 26^2}{26 \times 10900 (1 - {}^2/_{26})} = 2.46 \ sq\text{-}in$$

$$\bar{M}u = 156.000 + 2.46 \ \frac{10900}{20000} \times 5 \times 20.000 = 291.000$$

$$M_T = 1.208.000 + 291.000 = 1.499.000 \ in\text{-}lb$$

The calculation of A' s is not repeated because of the small differences found for the moment M_T

$$\rho = 1 - \frac{1}{1 - 0.403} \ \frac{5}{26} = 0.677$$

$$A_s = \frac{1.27}{0.677} \times 0.0091 \times 7.86 \times 26 + 2.46 \ \frac{10900}{20.000} \ \frac{1}{0.677} - \frac{16000}{20000} \ \frac{1}{0.677}$$

$$A_s = 3.48 \ sq\text{-}in$$

Trapezoidal and Rectangular Sections Combined

1st case: $\varkappa d < d_o$ (Fig. 10a)

$$v = \frac{1}{2}\left(1 + {}^{b_o}/_{b}\right) \ {}^{d_o}/_{d} + 1 - {}^{d_o}/_{d} \tag{84}$$

Coefficient $2\varkappa_1$

$$a = \frac{b_o}{b} + \frac{(1 - {}^{b_o}/_{b})\varkappa}{3} \tag{85}$$

$$2\varkappa_1 = \frac{a}{v} \tag{86}$$

Coefficient u:

$$u = \frac{2}{3}\left[1 - \frac{b_o/b + \frac{1 - b_o/b}{4}\kappa}{b_o/b + \frac{1 - b_o/b}{3}\kappa}\right] \tag{87}$$

2nd case: $\kappa d > d_o$ (Fig. 10b)

$$v = \frac{1}{2}\left(1 + b_o/b\right) d_o/d + 1 - d_o/d$$

$$b_i = vb \tag{88}$$

$$\alpha = b_o/b \left[b_o/b + \frac{(1 - b_o/b)}{3}\, d_o/\kappa d \right]$$

$$2\kappa_1 = \frac{\alpha}{v} \tag{89}$$

$$u = \frac{2}{3}\left[1 - \frac{b_o/b + \frac{1 - b_o/b}{4}\, d_o/\kappa d}{b_o/b + \frac{1 - b_o/b}{3}\, d_o/\kappa d}\right] \tag{90}$$

Coefficients are determined with the help of Tables V and VI

3rd case: (Fig. 10c)

$$v = \frac{1}{2}\left(1 + b_o/b\right) d_o/d + 1 - d_o/d$$

$$b_i = vb$$

$$2\kappa_1 = \frac{1}{v} \qquad u = 0$$

Example

For the section of Fig. 10b when given:

d = 24 in t = 26 in b = 12 in bo = 6 in do = 8 in

M = 600.000 in-lb N = 22.000 lb n = 15

fc = 900 lb/sq-in fs = 20.000 lb/sq-in

Determine A' s and As

$$Mu = 600\,000 + \frac{26}{2}\, 22000 = 886.000 \ in\text{-}lb$$

$$\kappa d = 0.403 \times 24 = 9.7 > d_o \qquad d_o/\kappa d = 8/9.7 = 0.825$$

$$b_o/b = \frac{6}{12} = 0.500 \qquad d_o/d = \frac{8}{24} = 0.333$$

$$v = 1/2\ (1+0.500)\ 0.333 + 1 - 0.333 = 0.817$$

$$b_i = 0.817 \times 12 = 9.8\ in$$

Tables V and VI give:

$$a = 0.630 \qquad u = 0.034$$

$$2x_1 = \frac{a}{v} = \frac{0.630}{0.817} = 0.772$$

$$K b_i d^2 = 157 \times 9.8 \times 24^2 = 885.000\ in\text{-}lb$$

$$\beta = 2 \times 20000\ \frac{20000}{900} \times 0.0091^2 = 73.7$$

$$M_u^o = 0.034 \times 73.7 \times 9.8 \times 24^2 = 14200\ in\text{-}lb$$

$$M_T = \frac{886.000}{0.772} + 14200 = 1.164.200\ in\text{-}lb$$

$$f_s' = 15 \times 900\ (1 - \frac{1}{0.403}\ \frac{2}{24}) = 10800\ lb/sq\text{-}in$$

$$A_s' = \frac{1.164.200 - 885\ 000}{24 \times 10800\,(1 - 2/24)} = 1.17\ sq\text{-}in$$

$$A_s = 0.772 \times 0.0091 \times 9.8 \times 24 + \frac{10800}{20000} \times 1.17 - \frac{22000}{20000}$$

$$A_s = 1.19\ sq\text{-}in$$

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T-SECTION

COEFFICIENT a

TABLE I

b_o/b \ d_o/kd	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
0.10	0.991	0.980	0.964	0.944	0.919	0.890	0.856	0.818	0.775	0.728	0.676	0.620	0.559	0.494
0.15	0.992	0.981	0.966	0.947	0.924	0.896	0.864	0.828	0.788	0.743	0.694	0.641	0.584	0.522
0.20	0.992	0.982	0.968	0.950	0.929	0.902	0.872	0.838	0.800	0.758	0.712	0.662	0.608	0.550
0.25	0.993	0.983	0.970	0.953	0.933	0.908	0.880	0.848	0.813	0.773	0.730	0.683	0.633	0.578
0.30	0.993	0.984	0.972	0.956	0.937	0.914	0.888	0.858	0.825	0.788	0.748	0.704	0.657	0.606
0.35	0.993	0.985	0.974	0.959	0.942	0.920	0.896	0.368	0.838	0.803	0.766	0.725	0.682	0.634
0.40	0.994	0.987	0.976	0.963	0.946	0.927	0.904	0.879	0.850	0.819	0.784	0.747	0.706	0.663
0.45	0.994	0.988	0.978	0.967	0.951	0.933	0.912	0.889	0.863	0.834	0.802	0.768	0.731	0.691
0.50	0.995	0.989	0.980	0.969	0.955	0.939	0.920	0.899	0.875	0.849	0.820	0.789	0.755	0.719

T-SECTION

COEFFICIENT u

TABLE II

b_o/b \ d_o/kd	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
0.10	-0.005	-0.011	-0.020	-0.029	-0.041	-0.053	-0.067	-0.081	-0.097	-0.112	-0.128	-0.143	-0.158	-0.171
0.15	-0.005	-0.011	-0.019	-0.028	-0.038	-0.050	-0.063	-0.076	-0.089	-0.104	-0.117	-0.131	-0.142	-0.153
0.20	-0.005	-0.011	-0.017	-0.026	-0.035	-0.047	-0.059	-0.071	-0.083	-0.096	-0.108	-0.119	-0.129	-0.137
0.25	-0.004	-0.010	-0.017	-0.025	-0.033	-0.044	-0.055	-0.066	-0.077	-0.088	-0.099	-0.109	-0.115	-0.122
0.30	-0.004	-0.009	-0.015	-0.023	-0.031	-0.041	-0.051	-0.061	-0.071	-0.081	-0.090	-0.098	-0.105	-0.109
0.35	-0.004	-0.009	-0.014	-0.021	-0.029	-0.038	-0.047	-0.056	-0.064	-0.074	-0.081	-0.089	-0.093	-0.097
0.40	-0.003	-0.007	-0.013	-0.019	-0.027	-0.034	-0.043	-0.050	-0.057	-0.066	-0.073	-0.079	-0.083	-0.085
0.45	-0.003	-0.007	-0.012	-0.017	-0.024	-0.031	-0.039	-0.046	-0.053	-0.059	-0.066	-0.070	-0.073	-0.074
0.50	-0.003	-0.006	-0.011	-0.016	-0.022	-0.028	-0.035	-0.041	-0.047	-0.053	-0.057	-0.062	-0.065	-0.065

TRIANGULAR AND RECTANGULAR SECTION

COEFFICIENT a

TABLE III

d_o/kd	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
α	0.370	0.390	0.413	0.437	0.463	0.490	0.519	0.550	0.583	0.617	0.653	0.690	0.729	0.770

TRIANGULAR AND RECTANGULAR SECTION

COEFFICIENT u

TABLE IV

d_o/kd	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
u	0.166	0.165	0.164	0.161	0.157	0.152	0.146	0.139	0.131	0.122	0.112	0.101	0.088	0.076

TRAPEZOIDAL AND RECTANGULAR SECTION

COEFFICIENT a

TABLE V

b_o/b \ d_o/kd	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
0.20	0.440	0.427	0.413	0.400	0.387	0.373	0.360	0.347	0.333	0.320	0.307	0.293	0.280	0.268
0.30	0.510	0.498	0.487	0.475	0.463	0.452	0.440	0.428	0.417	0.405	0.393	0.382	0.370	0.358
0.35	0.545	0.534	0.523	0.512	0.502	0.491	0.480	0.469	0.458	0.447	0.437	0.426	0.415	0.404
0.40	0.580	0.570	0.560	0.550	0.540	0.530	0.520	0.510	0.500	0.490	0.480	0.470	0.460	0.450
0.45	0.615	0.606	0.597	0.587	0.578	0.569	0.560	0.551	0.542	0.532	0.523	0.514	0.505	0.496
0.50	0.650	0.642	0.633	0.625	0.617	0.608	0.600	0.592	0.583	0.575	0.567	0.558	0.550	0.542
0.55	0.685	0.677	0.670	0.662	0.655	0.647	0.640	0.632	0.625	0.617	0.610	0.602	0.595	0.587
0.60	0.720	0.713	0.707	0.700	0.693	0.687	0.680	0.673	0.667	0.660	0.653	0.647	0.640	0.633
0.65	0.755	0.749	0.743	0.737	0.732	0.726	0.720	0.714	0.708	0.702	0.697	0.691	0.685	0.679
0.70	0.790	0.785	0.780	0.775	0.770	0.765	0.760	0.755	0.750	0.745	0.740	0.735	0.730	0.725
0.75	0.825	0.821	0.817	0.812	0.808	0.804	0.800	0.796	0.792	0.787	0.783	0.779	0.775	0.771
0.80	0.860	0.857	0.853	0.850	0.847	0.843	0.840	0.837	0.833	0.830	0.827	0.823	0.820	0.817

TRAPEZOIDAL AND RECTANGULAR SECTION

COEFFICIENT u

TABLE VI

b_o/b \ d_o/d	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25
0.20	0.092	0.089	0.085	0.083	0.081	0.077	0.074	0.071	0.066	0.063	0.058	0.052	0.047	0.045
0.30	0.069	0.066	0.065	0.061	0.058	0.057	0.053	0.049	0.047	0.043	0.039	0.036	0.031	0.027
0.35	0.059	0.057	0.055	0.053	0.051	0.048	0.045	0.043	0.039	0.037	0.033	0.029	0.026	0.023
0.40	0.052	0.050	0.047	0.045	0.043	0.041	0.037	0.036	0.033	0.031	0.028	0.025	0.022	0.019
0.45	0.045	0.043	0.041	0.039	0.037	0.035	0.033	0.031	0.028	0.026	0.023	0.021	0.018	0.015
0.50	0.039	0.037	0.035	0.033	0.031	0.029	0.028	0.026	0.024	0.022	0.019	0.017	0.015	0.013
0.55	0.033	0.031	0.030	0.028	0.027	0.025	0.023	0.022	0.020	0.018	0.017	0.015	0.013	0.011
0.60	0.028	0.027	0.025	0.024	0.023	0.021	0.019	0.018	0.017	0.015	0.013	0.012	0.011	0.009
0.65	0.024	0.022	0.021	0.020	0.019	0.017	0.016	0.015	0.013	0.012	0.011	0.010	0.009	0.007
0.70	0.019	0.018	0.017	0.016	0.015	0.014	0.013	0.012	0.011	0.010	0.009	0.008	0.007	0.006
0.75	0.015	0.015	0.013	0.013	0.012	0.011	0.010	0.009	0.009	0.008	0.007	0.006	0.005	0.005
0.80	0.011	0.011	0.011	0.010	0.009	0.009	0.008	0.007	0.007	0.006	0.005	0.005	0.004	0.003

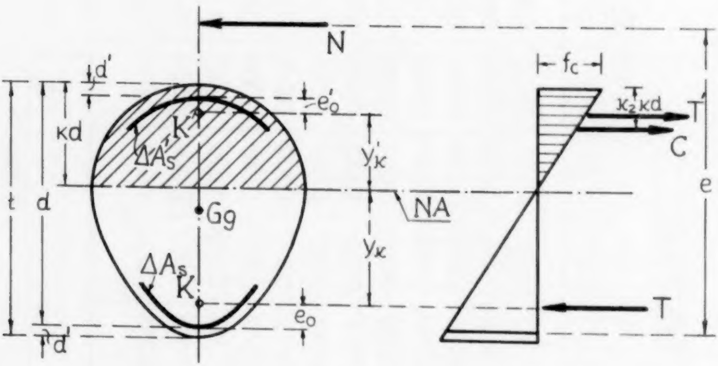

Fig. 1

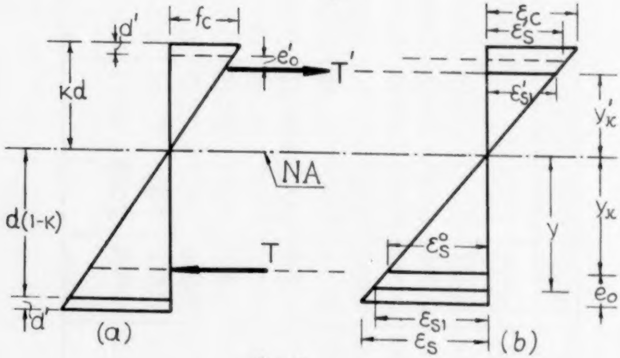

Fig. 2

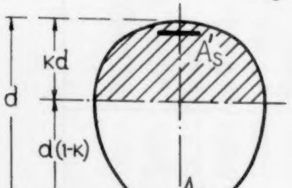

Fig. 3

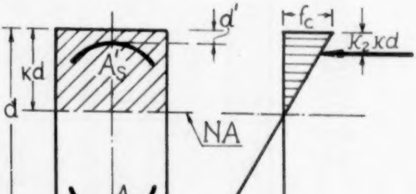

Fig. 4

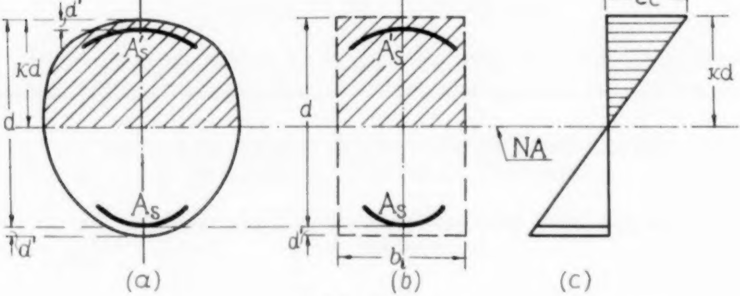

Fig. 5

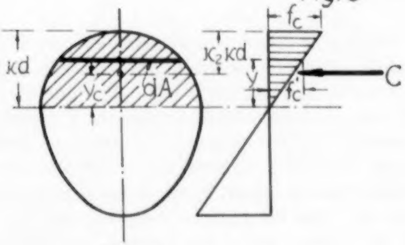

Fig. 6

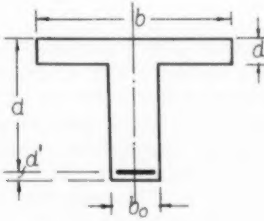

Fig. 7

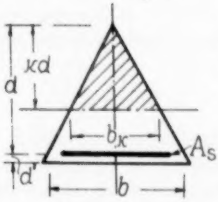

Fig. 8

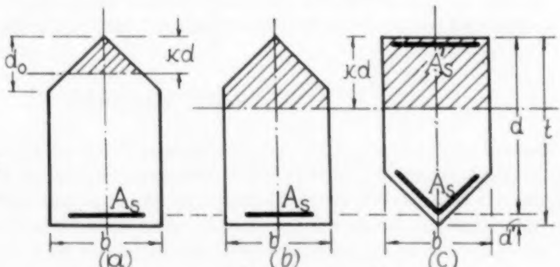

Fig 9

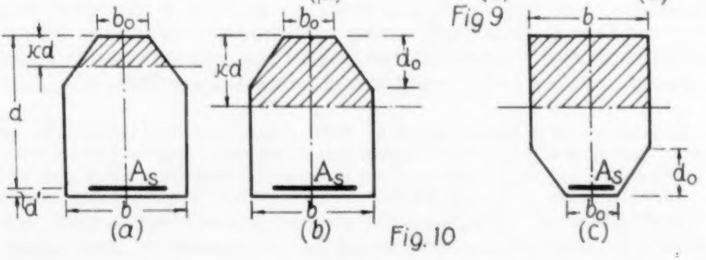

Fig. 10

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ULTIMATE STRENGTH ANALYSIS OF LONG HINGED REINFORCED CONCRETE COLUMNS

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(Proc. Paper 1510)

SYNOPSIS

Theoretical analyses are presented for the ultimate strength of long hinged reinforced concrete columns. Both concentric and eccentric loads are considered. The treatment of concentrically loaded columns is based on Engesser's tangent modulus formula and the treatment of eccentrically loaded columns is based on the principles advanced by Kármán. Both analyses utilize the stress-strain relationship for concrete determined by Hognestad from tests of short eccentrically loaded columns.

The analyses are compared with the test results of six experimental investigations carried out in the past. The test data, including 48 concentrically and 79 eccentrically loaded long hinged columns, and the analyses are in good agreement, thus indicating that the analyses with their basic assumptions yield reliable results.

INTRODUCTION

Extensive investigations of short concentrically and eccentrically loaded columns lead in the past to the development of generally applicable analytical expressions for predicting the ultimate strength of short reinforced concrete columns. The experimental studies have shown that for short columns the effect of lateral deflections on the strength is small and that short columns are not in danger of buckling. Accordingly, the analytical expressions disregard both deflections and buckling. On the other hand, it is known that the strength of long columns may be reduced significantly by both of these factors. The analyses presented in this paper take them into account.

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Reinforced concrete columns may fail either by material failure or by buckling. Material failure occurs by crushing of concrete in compression. Buckling occurs when the lateral deflection of the column increases without any increase of load.

Buckling of an eccentrically loaded hinged column is illustrated in Fig. 1 in which the moment resistance of a column and the external moment are shown as functions of the deflection at midheight. Figure 1 is drawn for a constant load P applied to a particular column at varying eccentricity. The moment resistance depends, therefore, only on the deflection of the column, while the external moment depends on the sum of the deflection and eccentricity.

The moment resistance is shown in Fig. 1 as the curve 1-2, the external moment as the straight line A-B for eccentricity e and as the straight line E-F for eccentricity e_{cr}; the two lines are parallel because their slope depends only on the magnitude of the load P which is constant. The straight line A-B intersects the curve 1-2 at points C and D for which the moment resistance is equal to the external moment. These two points represent two positions of equilibrium: stable equilibrium at C and unstable equilibrium at D. When the eccentricity of the load P is increased, points C and D move closer together. At the eccentricity e_{cr} line A-B becomes line E-F tangent to the curve 1-2. At an increase of the eccentricity beyond e_{cr} no position of equilibrium is possible. Thus, e_{cr} is the critical eccentricity corresponding to buckling of the column under the load P.

A column fails by material failure if the strength of the column material is exceeded before buckling is reached. For example, if the moment resistance curve in Fig. 1 ended at point C, the column would fail by material failure when the load P has reached eccentricity e.

NOTATION

The following notation is used throughout the paper:

A	= a parameter given in Table 1
A_s	= total area of longitudinal column reinforcement
B	= a parameter given in Table 1
b	= width of cross-section (Fig. 6)
d	= total depth of cross-section (Fig. 6)
d'	= effective depth of reinforcement (Fig. 6)
d''	= distance between centroids of compression and tension reinforcement (Fig. 6)
e	= end eccentricities of load for the case of $e_1 = e_2$ (Fig. 1)
e_1, e_2	= end eccentricities of load; e_2 is taken as the larger of the two values; e_2 is always taken as a positive quantity (Fig. 4)
e_{cr}	= critical eccentricity
E_c	= modulus of elasticity of concrete
E_s	= modulus of elasticity of steel

E_t = tangent modulus of elasticity

f_c = concrete stress

f'_c = compressive strength of 6 x 12-in. concrete cylinders

f''_c = compressive strength of concrete in flexure (Fig. 2)

f'_{cu} = compressive strength of 8-in. concrete cubes

f'_{pr} = compressive strength of concrete prisms

f_s = steel stress

f_{s2} = steel stress corresponding to strain ϵ_2

f_{s3} = steel stress corresponding to strain ϵ_3

f_y = yield point of reinforcement

I = moment of inertia

ℓ = length of column (Fig. 4)

ℓ_1, ℓ_2 = lengths defined in Fig. 4

L = equivalent length defined in Fig. 4

M = bending moment

M_{col} = moment resistance of a column

p = $\frac{A_s}{bd}$, ratio of reinforcement

P = applied load

P_t = critical load computed by the tangent modulus theory

x = distance in the direction of column axis from the point of maximum column deflection (Fig. 4)

y = lateral deflections of column including the initial eccentricity (Fig. 4)

y_m = maximum lateral deflection including the initial eccentricity (Fig. 4)

ϵ_2, ϵ_3 = steel strains (Fig. 6)

ϵ = strain

ϵ_1, ϵ_4 = concrete strains (Fig. 6)

ϵ_o = compressive strain corresponding to maximum concrete stress (Fig. 2)

ϵ_u = ultimate concrete strain in flexure (Fig. 2)

ϵ_y = yield point strain of reinforcement

$\frac{1}{\rho}$ = curvature

$\frac{1}{\rho_m}$ = curvature at $x = 0$

Ultimate Strength Analysis

Basic Assumptions

The assumptions for the theories presented in this paper were generally chosen so as to give a lower limit for the ultimate strength of both concentrically and eccentrically loaded long reinforced concrete columns.

Stress-strain relationship for concrete

The assumed stress-strain relationship for concrete in compression is shown in Fig. 2. It was derived by Hognestad[1] from the tests of short columns subjected to combined bending and axial load. It consists of a parabola and a sloping straight line.

The initial part of the curve in Fig. 2 is a second degree parabola expressed by the following equation:

$$f_c = f_c'' \left[\frac{2\epsilon}{\epsilon_o} - \left(\frac{\epsilon}{\epsilon_o}\right)^2 \right] \qquad (1)$$

The initial modulus of elasticity is taken as

$$E_c = 1,800,000 + 460\, f_c'' \,. \qquad (2)$$

The strain at the maximum stress is

$$\epsilon_o = 2\, f_c'' / E_c \,.$$

Between the strain ϵ_o and the ultimate strain ϵ_u, the stress-strain relationship is a descending straight line. The ultimate strain ϵ_u is taken as 0.0038.

The maximum stress f_c'' is taken as 0.85 times the cylinder strength f_c'. This relationship is believed to be correct only for vertically cast columns.

It is assumed further that concrete resists no tensile stresses. Since the concrete between the cracks carries some tensile stresses, the actual column deflections should be smaller than the computed ones. Thus, the assumption that no tensile stresses exist in the concrete should give somewhat lower ultimate loads than the actual.

The deflection of the column at buckling depends on the magnitude of strains in the concrete and steel. Since the buckling load depends on the deflection and since buckling can occur at almost any value of concrete strain, it is necessary to know the strains with a reasonable accuracy at all loads. Accordingly, a simplified stress block, such as a rectangle or a trapezoid, cannot yield generally accurate results and the use of a more complicated, but also more accurate, Hognestad's stress block is necessary.

Stress-strain relationship for reinforcing steel

The stress-strain diagram for the reinforcing steel is assumed trapezoidal with the flat portion equal to the yield point stress f_y as shown in Fig. 3. The modulus of elasticity, E_s, is taken in all calculations as 30,000,000 psi.

Bernoulli's hypothesis

Linear distribution of strains across the column cross-sections is assumed at all load levels.

Shape of deflected column

It is assumed that the deflected shape of a column is a part of a cosine wave as shown in Fig. 4. This assumption is theoretically correct only for small lateral deflections and for constant modulus of elasticity along the column. Such conditions are satisfied only in concentrically loaded columns.

For eccentrically loaded columns, the use of the cosine wave assumption implies that the modulus of elasticity is constant along the column and equal to the average modulus at the section of failure. The stiffness at the section of failure is the smallest one; therefore, the actual deflections are smaller than the computed ones and the computed ultimate loads should be somewhat smaller than the actual.

Effect of shear on deflection

For a solid cross-section, the effect of shear on deflections is small. It is disregarded in the theory presented herein.

Concentrically Loaded Columns

Buckling

A concentrically loaded long compression member remains straight as long as the load remains below the buckling load. When a column subjected to an axial load lower than the critical is deflected slightly from its straight position of equilibrium, its moment resistance increases faster than the corresponding external moment due to the lateral deflection and the column springs back to its straight position. At the buckling or critical load, the increase in external moment due to increase in lateral deflection of the column is equal to the corresponding increase in the moment resistance. The column deflects without further increase of load.

The buckling load for a concentrically loaded column stressed below the proportional limit of the material was determined theoretically by Euler[2,3] in 1744. For columns failing by buckling after the proportional limit has been exceeded, two modifications of the Euler's formula were proposed in 1889: Engesser's tangent modulus formula[4,5] and Considère's double modulus formula.[6] Shanley has shown by recent tests of aluminum columns[7,8,9] that the double modulus formula is based on an incorrect assumption and that the tangent modulus formula represents a lower limit for the buckling load of concentrically loaded columns.

It can be seen in Fig. 2 that the assumed stress-strain relationship for concrete has no proportional limit. Accordingly, the tangent modulus formula should be the most suitable for predicting the buckling strength of concentrically loaded reinforced concrete columns.

At any particular concentric load, the modulus of elasticity of the concrete and the moment of inertia are the same at all points. Accordingly, the following differential equation may be written for small lateral deflections:

$$-\frac{d^2y}{dx^2} E_t I = M \tag{3}$$

The solution of Eq. 3 gives the buckling load of a concentrically loaded column. The solution depends on the end conditions; for a column hinged at both ends it is

$$P_t = \frac{\pi^2 E_t I}{\ell^2} \tag{4}$$

where P_t is the buckling load, E_t the tangent modulus of elasticity of concrete at buckling, I the moment of inertia of the transformed cross-section of the column and ℓ the length of the column.

Material failure

An inspection of the assumed stress-strain relationships for concrete (Fig. 2) shows that a material failure occurs when the strain reaches the ultimate value for concrete $\epsilon_u = 0.0038$. In a concentrically loaded column, however, the material failure may be reached at a lower strain. For example, in a plain concrete column, the maximum material capacity corresponds to the strain ϵ_o; on the other hand, in a column reinforced with very large amount of steel having yield point larger than ϵ_o, the maximum material capacity is reached at yielding of the steel at a strain larger than ϵ_o. Therefore, the limiting value of the strain at failure of a concentrically loaded column depends on the strength of concrete, percentage of reinforcement and the yield point of reinforcement.

The material capacity of columns with reinforcement having a yield point lower than ϵ_o is always reached at the strain ϵ_o. For the practical range of concrete strengths, ϵ_o varies approximately from 1.2 to 2.3 percent. Thus for concentrically loaded columns reinforced with structural or intermediate grade steel, ϵ_o represents the limiting strain.

The material capacity of columns with reinforcement having a yield point higher than ϵ_o may be reached either at the strain ϵ_o or at a higher strain. Up to the strain of ϵ_o, the load carried by each material (i.e., steel and concrete) is increasing with increasing strain. After the strain ϵ_o is exceeded, however, the load carried by the concrete is decreasing while the load carried by the steel continues to increase. For low percentages of reinforcement, the loas in capacity of concrete cannot be compensated for by the increase of the load carried by the reinforcement so that material failure occurs at ϵ_o. On the other hand, for high percentages of steel, the loss in capacity of concrete is compensated for by the increase of the load carried by the reinforcement and the column is able to carry further increases of load beyond the strain ϵ_o.

It can be seen from the preceding discussion, that in most cases material capacity of concentrically loaded columns is reached at the strain ϵ_o. If present, the increase of capacity by straining the column beyond ϵ_o is usually small. It is assumed in the analysis, therefore, that for concentrically loaded columns ϵ_o represents the limiting value. In other words, if a concentrically loaded column does not buckle before reaching the unit shortening of ϵ_o, it fails by material failure.

Evaluation of ultimate load

The tangent modulus of elasticity E_t in Eq. 4 may be evaluated from the stress-strain diagram for concrete (Fig. 2) as $df_c/d\epsilon$. Therefore, as long as the strain ϵ at ultimate load does not exceed ϵ_o, the tangent modulus is given by the following equation:

$$E_t = E_c \left(1 - \frac{\epsilon}{\epsilon_o}\right) \tag{5}$$

where E_c is the initial modulus of elasticity given by Eq. 2.

For rectangular symmetrically reinforced concrete columns, the moment of inertia I of the transformed section may be written as:

$$I = \frac{bd^3}{12} + \frac{E_s p}{E_t} \frac{bd(d'')^2}{4} \qquad (6)$$

where E_s/E_t is the modular ratio; the modular ratio varies with the load. For strains smaller than the yield point strain of the reinforcement ϵ_y, the modulus of elasticity E_s may be taken as 30×10^6 psi. For strains equal and larger than the yield point strain ϵ_y, E_s is equal to zero. Accordingly, the moment of inertia I decreases suddenly when the yield point strain is reached.

For strains smaller than ϵ_o, the relationship between the concentric load and the corresponding strains may be expressed as

$$\frac{P}{f''_c bd} = \frac{f_s p}{f''_c} + \frac{2\epsilon}{\epsilon_o} - \left(\frac{\epsilon}{\epsilon_o}\right)^2 \qquad (7)$$

where f_s is equal to $E_s \epsilon$ for strains smaller than ϵ_y, and equal to f_y for strains equal or larger than ϵ_y.

Equations 4, 5, 6 and 7 contain four unknowns: P, E_t, I and ϵ. Thus, the buckling load may be evaluated by simultaneous solution of all four equations. If the resulting ϵ is larger than ϵ_o, the strength of the column is governed by material failure and its capacity may be evaluated from Eq. 7 by substituting $\epsilon = \epsilon_o$. Similarly, the critical length ℓ for a column of known cross-section and a known value of the load P may be found by a simultaneous solution of Eq. 4-7 for ℓ, E_t, I and ϵ.

Eccentrically Loaded Columns

Buckling

Under an eccentric loading, a column deflects laterally; the lateral deflection increases with the load. If the stresses in the column remain below the proportional limit, at large deflections the load approaches the value of the buckling load for the same column loaded concentrically.

A general theory for determining the buckling load of an eccentrically loaded column in which the maximum stresses have exceeded the proportional limit of the material was proposed by Kármán(10) in 1910. The theory is based on the stress-strain relationship of the column material and on the assumption of linear strain distribution. The deflected shape of the column is determined by numerical integration of angle changes along the column length. The validity of Kármán's theory was proved by tests of hinged rectangular steel columns reported by Chwalla(11) in 1934.

The determination of the exact shape of the deflected column by numerical integration is very laborious. Roš and Brunner(12) simplified Kármán's theory by assuming the deflected column shape as a half sine wave. Westergaard and Osgood(13) proposed the use of a part of a cosine wave as the deflected column shape. The sine wave assumption is inconsistent with the actual conditions in that it corresponds to zero curvature, i.e., an infinite stiffness, at the ends of the column. The cosine wave assumption is

theoretically correct only at small lateral deflections as long as the stresses in the column remain below the proportional limit. When the proportional limit is exceeded, the cosine wave assumption gives results on the safe side.

The basic work on buckling of eccentrically loaded reinforced concrete columns was carried out by Baumann(14) in 1930-33. Using Kármán's procedure in conjunction with experimentally determined properties of concrete, he found a good agreement between the tests and the theory. More recently, Ernst, Hromadik and Riveland(15) applied to reinforced concrete columns Kármán's method as modified by Westergaard and Osgood, limiting themselves to hinged columns with equal eccentricities at both column ends and using Hognestad's stress-strain relationship for concrete. The method presented herein is essentially the same as that presented by Ernst, Hromadik and Riveland but it is applicable to hinged columns both with equal and unequal eccentricities at the column ends.

An eccentrically loaded column becomes unstable when one of the following conditions is satisfied:

$$\frac{de}{dy_m} = 0, \quad \frac{dP}{dy_m} = 0, \quad \text{or} \quad \frac{dM}{dy_m} = 0 .$$

If the relationship between the load P, moment M and strains is unique, as is assumed in this derivation, all three conditions give the same buckling load.

In the following, the eccentricities at the ends of the column are expressed in terms of the maximum total deflection y_m. The buckling load is then computed from the condition $de/dy_m = 0$.

A deflected hinged column is shown schematically in Fig. 4. Since it is assumed that the deflected shape of the eccentrically loaded column is a part of a cosine wave, it follows that:

$$y = y_m \cos \frac{\pi x}{L} \tag{8}$$

The eccentricities at the column ends are then expressed as:

$$e_1 = y_m \cos \frac{\pi \ell_1}{L} \tag{9}$$

and

$$e_2 = y_m \cos \frac{\pi \ell_2}{L} \tag{10}$$

Observing that $\ell_1 + \ell_2 = \ell$, rearranging and adding of Eq. 9 and 10 gives:

$$\frac{\pi}{L} = \frac{1}{\ell} \left(\text{arc cos} \frac{e_1}{y_m} + \text{arc cos} \frac{e_2}{y_m}\right) \tag{11}$$

The first derivative of Eq. 11 with respect to y_m includes expressions de_1/dy_m and de_2/dy_m which are equal to zero at buckling. After substituting from Eq. 9 and 10, the derivative of Eq. 11 may, therefore, be written as:

$$\frac{d(\frac{\pi}{L})}{dy_m} = \frac{1}{\ell y_m}(\cot\frac{\pi\ell_1}{L} + \cot\frac{\pi\ell_2}{L})$$

or as

$$\frac{y_m}{\frac{\pi}{L}}\frac{d(\frac{\pi}{L})}{dy_m} = \frac{\cot\frac{\pi\ell_1}{L} + \cot\frac{\pi\ell_2}{L}}{\frac{\pi\ell}{L}} \tag{12}$$

Solution of Eq. 12 gives the critical slenderness ℓ, at which a column will buckle under an assumed load, and the corresponding values ℓ_1 and $\ell/_2$. The corresponding end eccentricities may then be computed from Eq. 9 and 10.

A graphical solution of Eq. 12 is shown in Fig. 5. In Fig. 5a, equation 12 is solved for several values of $\pi\ell/L$ and ℓ_1/ℓ. In Fig. 5b, equations 9 and 10 are solved for several values of the same two quantities. Fig. 5a and 5b are combined into Fig. 5c and 5d relating the left side of Eq. 12 to $\pi\ell/L$ and e_2/y_m, respectively, for several ratios of e_1/e_2.

The left side of Eq. 12 may be evaluated from the relationships between the curvature and the deflection of the column, and between the axial load, moment and deflection. The curvature and the maximum total deflection are related as follows:

$$\frac{1}{\rho} = -\frac{d^2y}{dx^2} = y_m(\frac{\pi}{L})^2 \cos\frac{\pi x}{L} \tag{13}$$

At the point of maximum moment

$$\frac{1}{\rho_m} = y_m(\frac{\pi}{L})^2 \tag{14}$$

Since the curvature may be expressed also as

$$\frac{1}{\rho_m} = \frac{\epsilon_4 - \epsilon_1}{d}$$

Eq. 14 may be written in the following form

$$\frac{\epsilon_4 - \epsilon_1}{d} = y_m(\frac{\pi}{L})^2 \tag{15}$$

where ϵ_4 and ϵ_1 are the strains at the section of maximum moment and d is the depth of the column (Fig. 6).

At the point of maximum moment, the relation between the load, moment and maximum total deflection may be expressed as

$$y_m = \frac{M_{col}}{P} \tag{16}$$

The relationship between P, M_{col} and the strains may be determined from statics and the stress-strain relationship for the particular material as is shown in a later part of this paper.

The critical eccentricity and the length of the column, for which an assumed load is the buckling load, may be determined from Eq. 15 and 16 with the aid of Fig. 5c,d. For a constant value of P, several corresponding values of $(\epsilon_4 - \epsilon_1)$ are chosen and the corresponding values of M_{col} are computed. For each value of $(\epsilon_4 - \epsilon_1)$ the deflection y_m is determined from Eq. 16 and the value of π/L is determined from Eq. 15. For each set of corresponding values of y_m and π/L, the quantity

$$\frac{y_m}{\frac{\pi}{L}} \quad \frac{d(\frac{\pi}{L})}{dy_m}$$

is computed. Finally, Fig. 5c and 5d are entered with the vertical ordinate, and the critical length ℓ and eccentricity e_2 are evaluated from the horizontal ordinates.

If dimensionless ratios y_m/d, $\pi d/L$, $M_{col}/f_c''bd^2$ and $P/f_c''bd$ are used instead of y_m, π/L, M_{col} and P, respectively, the method of solution is unaffected since the right side of Eq. 12 remains unchanged. However, the results are obtained in terms of ratios ℓ/d and e_2/d instead of ℓ and e_2.

The equations derived above are a general solution for buckling of eccentrically loaded hinged columns. They are applicable to columns of any cross-section and of any material.

Relationships between loads, moments and strains

The distribution of stresses in a reinforced concrete column is determined by two strains, the stress-strain relationships for concrete (Fig. 2) and steel (Fig. 3), and the cross-section of the column. The load P and moment M_{col} are expressed in terms of strains by summation of the stresses in the cross-section.

For rectangular cross-sections with symmetrical reinforcement illustrated in Fig. 6, the following equations may be derived:

$$\frac{P}{f_c'' bd} = \frac{p(f_{s2} + f_{s3})}{2 f_c''} + \frac{A}{\epsilon_4 - \epsilon_1} \qquad (17)$$

$$\frac{M_{col}}{f_c'' bd^2} = \frac{p(f_{s3} - f_{s2})}{4 f_c''} \frac{d''}{d} + \frac{B}{(\epsilon_4 - \epsilon_1)^2} - \frac{\epsilon_4 + \epsilon_1}{(\epsilon_4 - \epsilon_1)^2} \frac{A}{2} \qquad (18)$$

where parameters A, B and steel stresses f_{s2} and f_{s3} are known functions of concrete strains ϵ_1 and ϵ_4.

Formulas for parameters A and B are listed in Table 1. Four expressions are given for each parameter; each formula is applicable within certain limits for ϵ_4 and ϵ_1. Formulas for cases 1 and 3 refer to full cross-section in compression, while formulas for cases 2 and 4 refer to a cross-section partly in compression and partly in tension.

The steel stresses are related to steel strains ϵ_2 and ϵ_3 as $f_{s2} = E_s \epsilon_2$ and $f_{s3} = E_s \epsilon_3$, and strains ϵ_2 and ϵ_3 may be expressed as

$$\epsilon_2 = \epsilon_4 - (\epsilon_4 - \epsilon_1)\frac{d'}{d} \tag{19}$$

$$\epsilon_3 = \epsilon_1 + (\epsilon_4 - \epsilon_1)\frac{d'}{d} \tag{20}$$

It should be noted from Fig. 3 that the steel stresses f_{s2} and f_{s3} have an upper limit equal to the yield point stress f_y.

In the derivation of Eq. 17 and 18, compressive strains were taken as positive quantities and tensile strains as negative quantities. The same sign convention should be followed in using these equations.

Material failure

An eccentrically loaded column fails by crushing of concrete when the maximum strain ϵ_4 reaches the ultimate value for concrete ϵ_u. The load and moment at failure of such a column may be computed by solution of Eq. 9, 10, 15, 16, 17, 18, 19 and 20 after placing $\epsilon_4 = \epsilon_u$. This solution differs from the well-known equations for short columns(1) only in that it accounts automatically for the deflections of the column.

Graphs of Ultimate Loads

The procedure for eccentrically loaded columns described in the preceding section does not lend itself easily to a direct evaluation of the buckling load for a particular column. It is more suitable for correlating the ultimate loads to the critical lengths and eccentricities for a whole group of columns having the same properties of the cross-section and the same characteristics of loading other than the magnitude of the initial eccentricity. The ultimate load for a particular column of the group may then be obtained by interpolation.

It is not necessary to make a separate set of calculations for each particular column cross-section. If dimensionless quantities are used instead of the actual loads, moments and dimensions, one group pertains to all columns having the same properties of concrete, properties of steel, percentage of reinforcement, effective depth ratio d'/d and conditions of loading. For convenient use, the results may then be tabulated for each group or plotted as is illustrated in Fig. 7-10.

Twenty-eight dimensionless graphs were prepared for rectangular columns with symmetrical reinforcement; eight representative graphs are included in Fig. 7-10. The ultimate load reduced to the dimensionless quantity $P/f_c''bd$ is plotted as a function of the slenderness ratio ℓ/d for initial eccentricity ratios e_2/d ranging from 0 to 1.0. The slenderness ratio is covered in each graph in the range of 0 to 60. For $f_y = 50{,}000$ psi, $e_1/e_2 = 1.0$ and $d'/d = 0.9$, concrete strengths $f_c' = 2000$, 3000, 4000, 6000 psi and steel percentages $p = 1.0$, 2.0, 4.0 percent were included in all combinations, thus, giving 12 graphs. The selection of the characteristic quantities for the remaining 16 graphs was aimed in part at checking the theoretical analysis against the available test data and in part at evaluating the effects of several variables.

Effects of Variables

The eight representative graphs included in Fig. 7 through 10 illustrate the effects of slenderness ratio, eccentricity ratio, concrete strength, percentage

of reinforcement, yield point of reinforcement, ratios of end eccentricities and long duration of loading. An inspection of the graphs shows that the slenderness ratio ℓ/d and the eccentricity ratio e_2/d are the most important variables. An increase of ℓ/d from 0 to 60 or an increase of e_2/d from 0 to 1.0 may reduce the ultimate load more than 80 percent. The effects of concrete strength f'_c and of the percentage of reinforcement p are illustrated in Fig. 7 and 8, respectively; it can be seen that the ultimate load increases with both of these variables less than in direct proportion. It may be also noted that the effects of e_2/d, f'_c and p on the strength of a long column are of about the same order of magnitude as for a short column.

The effect of the yield point of reinforcement f_y on the ultimate strength depends on the extent of yielding indicated in Fig. 9a by the dotted lines dividing the graph into five areas. In area I, the steel stresses at failure are below the yield point; thus, the yield point has no effect. In area II, all steel yields in compression; in area III, the steel yields in compression on one face of the column; in area IV, the steel yields in compression on one face and in tension on the other face; finally, in area V, the steel yields in tension on one face. It can be seen by comparing Fig. 9a and 9b, that in areas II - V, the ultimate load increases with increasing yield point.

The effect of the ratio of the end eccentricities can be seen by comparing Fig. 9b and 10a. The ultimate load is smallest for $e_1 = e_2$ and largest for $e_1 = -e_2$. In short columns with unequal eccentricities, there is no decrease in ultimate load with the increase of slenderness because such columns fail at the end sections for which the eccentricity at failure is equal to the initial eccentricity. The range of slenderness for which the ultimate load is constant increases with decreasing e_1/e_2 ratios and with increasing eccentricities.

The graphs in Fig. 7, 8, 9 and 10a represent the strength of columns subjected to short-time loading.* Earlier tests have shown that the maximum load which a short eccentrically loaded column can maintain indefinitely is about 10 percent lower than the corresponding short-time strength. The primary effects of creep appear to be a substantial increase of the maximum concrete strain at failure ϵ_u and a relatively small decrease of the maximum concrete stress f''_c.(16) Neglecting the small effect of creep on concrete strength, the stress-strain relationship for long-time loading may be taken as shown in Fig. 2 but with the strain ordinates multiplied by a factor of 2. The ultimate loads may then be computed in the same manner as for short-time loading; Fig. 10b represents an ultimate load graph computed in this manner. A comparison of this graph with Fig. 9b illustrates the difference between the maximum load which a column can maintain for a short time and that which can be sustained indefinitely.

A quantitative investigation was made also of the effect of the effective depth ratio d'/d. A decrease of this ratio from 0.9 to 0.8 was found to decrease the ultimate load as much as 10 percent.

Experimental Check of Analyses

The theoretical analyses of the strength of concentrically and eccentrically loaded long reinforced concrete columns presented in this paper are based on a set of assumptions derived from several investigations of related problems.

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* A test to failure in a conventional testing machine carried out in a few hours.

To check the accuracy of the analyses, the theoretical ultimate loads were compared with the results of tests of 48 concentrically and 79 eccentrically loaded long hinged columns. The test data were obtained in six independent investigations reported by Baumann[14] in 1934, by Thomas[17] in 1939, by Hanson and Rosenström[18] in 1947, by Rambøll[19] in 1951, by Ernst, Hromadik and Riveland[15] in 1953 and by Gehler and Hütter[20] in 1954. In addition, the analysis was checked against the tests of short columns reported by Hognestad[1] in 1951.

Experimental Data

All six investigations of long columns were carried out with square or rectangular tied columns reinforced with four symmetrically placed bars. The smallest cross-section was 3 x 3 in. and the largest one was 10 x 10 in. The length varied from 74 to 252 in. The concrete was made of Portland cement, sand and gravel. Some of the columns were cast in a horizontal, others in a vertical, position. The strength of concrete was determined by tests of cubes, prisms or cylinders cast, cured and tested together with the columns. In the investigation reported by Hanson and Rosenstrom, control prisms were cut out of the undistrubed ends of the columns after the test because control cubes indicated an unexpectedly low strength. The tests by Ernst, Hromadik and Riveland were the only investigation in which the strength of concrete was determined from 6 x 12-in. cylinders.

In Rambøll's and some of Baumann's tests, columns were tested in a horizontal position, in those cases, the effect of dead load was counteracted by a suspension system. The other columns were tested in a vertical position. In all tests, the load was applied through hinges usually made of knife edges.

In most eccentrically loaded columns, the load was applied at a certain eccentricity determined by direct measurements.* Of these, in 52 columns, the eccentricities were equal at both column ends ($e_1/e_2 = 1.0$) and in three columns tested by Baumann the eccentricity at one end was equal to zero ($e_1/e_2 = 0$). In 24 tests by Gehler and Hütter, the "eccentrically loaded" columns were subjected to a concentric axial load P and a transverse load H applied at the middepth of the specimen. The horizontal force was kept at one, two or three percent of the vertical load, so that the "initial eccentricity" at middepth was constant throughout the test and equal to

$$e = \frac{H\ell}{4P}$$

The test data provide a wide coverage of six major variables. The following ranges were covered by the 127 long columns:

1. Slenderness, $\ell/d = 11.7 - 40.7$
2. Concrete strength, $f'_c = 2130 - 6900$ psi
3. Ratio of reinforcement, $p = 0.005 - 0.050$
4. Yield point of reinforcement, $f_y = 28.9 - 53.8$ ksi
5. Effective depth, $d'/d = 0.77 - 0.91$
6. Eccentricity, $e_2 = 0 - 0.833$

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* In Thomas' tests, the initial eccentricities were determined also from strain and deflection measurements, but the directly measured initial eccentricities were used in this study.

Both the concentrically and eccentrically loaded columns covered separately almost complete ranges of variables 1 through 5. The minimum eccentricity for the eccentrically loaded columns was $e_2/d = 0.005$.

From the numerous existing tests of short columns, Series II and III of Hognestad's investigation(1) were selected for checking the equations presented in this paper. Columns in both series had a rectangular cross-section 10 x 10 in. reinforced symmetrically with 2.4 and 4.8 percent of longitudinal bars, respectively. All columns were 75 in. long and had equal end eccentricities, i.e., $e_2 = e_1$. The eccentricities varied from 0 to 12.5 inches. All columns were cast and tested in the same vertical position. The strength of concrete was determined by tests of 6 x 12-in. control cylinders.

The slenderness and eccentricity ratios and the ultimate loads reduced to the dimensionless quantity $P_{test}/f''_c bd$ are listed in Table 2 for short columns, in Table 3 for long concentrically loaded columns and in Table 4 for long eccentrically loaded columns.

Calculated Ultimate Loads

Tables 2, 3 and 4 contain also calculated ultimate loads reduced to the dimensionless quantity $P_{calc}/f''_c bd$. It was assumed in computing the theoretical loads for short concentrically loaded columns that the strain at failure is equal to ϵ_o and that the steel stress at failure f_s is equal to the yield point of reinforcement f_y. The ultimate loads were then computed directly from Eq. 7. In computing the ultimate loads for the short eccentrically loaded columns, the maximum strain in concrete at failure ϵ_4 was assumed equal to $\epsilon_u = 0.0038$. Setting $e_1 = e_2 = e$ and $\ell_1 = \ell_2 = \ell/2$, the eccentricity of load may be expressed from Eq. 9, 15 and 16 as

$$e = \frac{M_{col}}{P} \cos\left[\frac{\ell}{2d}\sqrt{(0.0038 - \epsilon_1)\frac{Pd}{M_{col}}}\right] \tag{21}$$

For several chosen values of ϵ_1 and f'_c, the ratio M_{col}/P was computed from Eq. 17, 18, 19 and 20. The corresponding eccentricities e causing failure were then computed from Eq. 21. The resulting values were plotted in graphs of $P_{calc}/f'_c bd$ vs. e/d for each series as functions of the concrete strength f'_c. The ultimate loads for the test columns were obtained by entering the graphs with the known eccentricity and concrete strength. It should be noted that the eccentricity e is the eccentricity at the end of the column and, thus, not subject to change during the test.

The theoretical ultimate loads for long concentrically loaded columns were calculated directly from Eq. 4, 5, 6 and 7 for each test column.

The theoretical ultimate loads for long eccentrically loaded columns were obtained from the graphs of ultimate loads. This procedure involved several interpolations and for some columns also extrapolations; therefore, the resulting $P_{calc}/f''_c bd$ values in Table 4 are given only to two decimal places.

The calculated loads are based on the dimensions and material properties of the test columns as reported in the respective test reports. Data not given in the reports were estimated; of these, the most important were the assumption concerning the relationships between the strength of concrete in the columns, cylinders, prisms and cubes.

The stress-strain relationship assumed in the analysis for concrete is

based on the strength f''_c. In computing the ultimate loads for vertically cast columns, this quantity was assumed related to the strength of 6 x 12-in. control cylinders as $f''_c = 0.85 f'_c$. For horizontally cast columns, the relationship $f''_c = f'_c$ was assumed as indicated by earlier tests.[16] Where cylinder strengths were not given, the following relationships were assumed between the strength of cylinders, prisms and cubes: $f'_c = f'_{pr}$ and $f'_c = 0.8 f'_{cu}$.

Analyses vs. Tests

The ratios of the test to calculated ultimate loads for short columns are listed in the last column of Table 2. For all short concentrically loaded columns, the ratio is equal or smaller than 1.0 and, thus in agreement with Hognestad's observation of the presence of small accidental eccentricities.[1] The average ratio for the short concentrically loaded columns is 0.901 and the standard deviation is 0.060. For all short eccentrically loaded columns, the ratios are close to 1.0; the over-all average is 1.000 and the standard deviation is 0.058. It should be noted, however, that a good over-all agreement between the theory and the Hognestad's test data should be expected in view of the fact that the properties of the concrete stress block shown in Fig. 2 were derived from the same investigation.[1] On the other hand, the excellent agreement from group to group and for all individual columns is significant in judging the correctness of the theoretical equations and of the basic assumptions.

The analysis of the ultimate strength of long columns is compared with the test data in the last columns of Tables 3 and 4 and in Fig. 11 and 12 giving the ratio P_{test}/P_{calc} for every column. An examination of the tables and figures shows that the scatter is noticeably larger for the concentrically loaded columns; that of the 127 columns tested, six concentrically loaded columns were more than 20 percent weaker and none of the eccentrically loaded columns was more than 11 percent weaker than predicted by the theory; that the average ratio is higher for the eccentrically than for the concentrically loaded columns; and that no definite trends exist with any of the six major variables except as mentioned above.

The comparisons are summarized in Table 5. The average values for the individual investigations show that in all but one case the average is smaller for the concentric than for the eccentric loading. This result is identical with that for short columns tested by Hognestad, thus indicating that it is difficult to obtain concentric hinged loading. Furthermore, the averages for eccentrically loaded columns of every investigation exceed 1.00; this agrees with the fact that several assumptions of the theory represent a minimum condition as far as the ultimate strength is concerned. Finally, the higher over-all standard deviation for concentrically loaded columns reemphasizes the smaller spread in the P_{test}/P_{calc} ratios for eccentrically loaded columns. The poorer agreement for the concentrically loaded columns is understandable since differences and small unavoidable errors in testing techniques affect the test value of ultimate load more for a concentrically than for an eccentrically loaded column.

The over-all average value of the ratio of measured to computed ultimate loads of long columns is 1.066 and the standard deviation is 0.153. This, together with the reasonably good agreement for the individual investigations and absence of any definite trends with any major variable, loads to the conclusion that the ultimate load of hinged columns can be predicted accurately by the analyses presented in this paper.

CONCLUDING REMARKS

The ultimate load analyses presented herein were aimed primarily at improving the knowledge of the effect of deflections and buckling on the strength of reinforced concrete columns and to provide a tool for further studies.

The good agreement with the available test data indicates that the analyses are based on reasonable assumptions. Particularly, it serves as a further proof that Hognestad's stress block approximates well the actual stress-strain relationship for concrete subjected to combined axial compression and bending.

As a tool for further studies, the analyses permit the evaluation of the effects of various variables on the column strength. Such studies are indispensable in preparation of simplified design procedures.

And finally, the analyses permit construction of ultimate load tables and graphs, which can be used conveniently whenever it is necessary to evaluate accurately the ultimate loads of hinged reinforced concrete columns.

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TABLE 1

RELATIONSHIP BETWEEN LOAD, MOMENT AND STRAINS FOR SYMMETRICAL RECTANGULAR CROSS-SECTIONS OF REINFORCED CONCRETE

$$\frac{P}{f_c'' bd} = \frac{p(f_{s2}+f_{s3})}{2f_c''} + \frac{A}{(\epsilon_4-\epsilon_1)} \qquad \frac{M_{col}}{f_c'' bd^2} = \frac{p(f_{s3}-f_{s2})}{4f_c''}\frac{d''}{d} + \frac{B}{(\epsilon_4-\epsilon_1)^2} - \frac{(\epsilon_4+\epsilon_1)}{(\epsilon_4-\epsilon_1)^2}\frac{A}{2}$$

Case 1 $\epsilon_4<\epsilon_o$ $\epsilon_1>0$	$A = \frac{(\epsilon_4^2-\epsilon_1^2)}{\epsilon_o} - \frac{(\epsilon_4^3-\epsilon_1^3)}{3\epsilon_o^2}$ $\quad B = \frac{2(\epsilon_4^3-\epsilon_1^3)}{3\epsilon_o} - \frac{(\epsilon_4^4-\epsilon_1^4)}{4\epsilon_o^2}$
Case 2 $\epsilon_4<\epsilon_o$ $\epsilon_1<0$	$A = \frac{\epsilon_4^2}{\epsilon_o} - \frac{\epsilon_4^3}{3\epsilon_o^2}$ $\quad B = \frac{2\epsilon_4^3}{3\epsilon_o} - \frac{\epsilon_4^4}{4\epsilon_o^2}$
Case 3 $\epsilon_o<\epsilon_4<\epsilon_u$ $\epsilon_1>0$	$A = \frac{2\epsilon_o^3-3\epsilon_o\epsilon_1^2+\epsilon_1^3}{3\epsilon_o^2} + \frac{(\epsilon_u-0.85\epsilon_o)(\epsilon_4-\epsilon_o)-0.075(\epsilon_4^2-\epsilon_o^2)}{(\epsilon_u-\epsilon_o)}$ $B = \frac{2(\epsilon_o^3-\epsilon_1^3)}{3\epsilon_o} - \frac{(\epsilon_o^4-\epsilon_1^4)}{4\epsilon_o^2} + \frac{(\epsilon_u-0.85\epsilon_o)(\epsilon_4^2-\epsilon_o^2)-0.10(\epsilon_4^3-\epsilon_o^3)}{2(\epsilon_u-\epsilon_o)}$
Case 4 $\epsilon_o<\epsilon_4<\epsilon_u$ $\epsilon_1<0$	$A = \frac{2}{3}\epsilon_o + \frac{(\epsilon_u-0.85\epsilon_o)(\epsilon_4-\epsilon_o)-0.075(\epsilon_4^2-\epsilon_o^2)}{(\epsilon_u-\epsilon_o)}$; $\quad B = \frac{5\epsilon_o^2}{12} + \frac{(\epsilon_u-0.85\epsilon_o)(\epsilon_4^2-\epsilon_o^2)-0.10(\epsilon_4^3-\epsilon_o^3)}{2(\epsilon_u-\epsilon_o)}$

Note: Compressive strains should be taken as positive quantities, tensile strains as negative quantities. The steel stresses $f_{s2} = E_s\epsilon_2$ and $f_{s3} = E_s\epsilon_3$ have an upper limit equal to the yield point stress f_y.

TABLE 2

HOGNESTAD'S TESTS OF SHORT COLUMNS (1)

$b \times d$ = 10 in. x 10 in., ℓ/d = 7.5

Specimen	Concrete Strength f'_c, psi.	Eccentricity e_2/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
		Series II			
B-6a	4080	0	1.314	1.312	1.00
b	4040	0	1.224	1.315	0.93
C-6a	2020	0	1.308	1.630	0.80
b	1520	0	1.566	1.836	0.85
			Average Col. 6		0.90
A-7a	5240	0.325	0.515	0.553	0.93
b	5810	0.25	0.575	0.622	0.92
B-7a	4080	0.25	0.738	0.689	1.07
b	4040	0.25	0.723	0.692	1.04
C-7a	1970	0.25	0.844	0.881	0.96
b	1520	0.25	0.983	0.922	1.07
			Average Col. 7		1.00
A-8a	5520	0.50	0.345	0.359	0.96
b	5810	0.50	0.308	0.346	0.89
B-8a	4700	0.50	0.390	0.395	0.99
b	4260	0.50	0.403	0.414	0.97
C-8a	1820	0.50	0.639	0.612	1.04
b	1820	0.50	0.639	0.620	1.03
			Average Col. 8		0.98
A-9a	5100	0.75	0.205	0.211	0.97
b	5170	0.75	0.208	0.208	1.00
B-9a	4700	0.75	0.235	0.226	1.04
b	4370	0.75	0.241	0.238	1.01
C-9a	1880	0.75	0.456	0.446	1.02
b	1730	0.75	0.446	0.460	0.97
			Average Col. 9		1.00
A-10a	5100	1.25	0.106	0.102	1.04
b	5170	1.25	0.100	0.101	0.99
B-10a	4260	1.25	0.120	0.121	0.99
b	4370	1.25	0.119	0.119	1.00
C-10a	2300	1.25	0.227	0.222	1.02
b	1770	1.25	0.300	0.252	1.19
			Average Col. 10		1.04
			Average Series II		0.989
			Standard Deviation		0.073

TABLE 2 (Concluded)

Specimen	Concrete Strength f'_c, psi.	Eccentricity e_2/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
		Series III			
B-11a	3870	0	1.520	1.636	0.93
b	4010	0	1.422	1.614	0.88
C-11b	2070	0	2.006	2.189	0.92
			Average Col. 11		0.91
A-12a	4150	0.25	0.892	0.848	1.05
b	5050	0.25	0.758	0.792	0.96
B-12a	4300	0.25	0.828	0.839	0.99
b	4010	0.25	0.833	0.856	0.97
C-12a	2300	0.25	1.286	1.147	1.12
b	2200	0.25	1.230	1.163	1.06
			Average Col. 12		1.03
A-13a	5350	0.50	0.484	0.514	0.94
b	4850	0.50	0.510	0.541	0.94
B-13a	3580	0.50	0.592	0.639	0.93
b	4290	0.50	0.564	0.571	0.99
C-13a	2300	0.50	0.770	0.800	0.96
b	2070	0.50	0.778	0.829	0.94
			Average Col. 13		0.95
A-14a	5350	0.75	0.312	0.339	0.92
b	5100	0.75	0.353	0.353	1.00
B-14a	3580	0.75	0.457	0.462	0.99
C-14a	1950	0.75	0.696	0.655	1.06
b	2070	0.75	0.591	0.641	0.92
			Average Col. 14		0.98
A-15a	5100	1.25	0.203	0.184	1.10
b	4850	1.25	0.192	0.193	0.99
B-15a	3800	1.25	0.229	0.245	0.93
b	4630	1.25	0.214	0.201	1.06
C-15a	1950	1.25	0.437	0.426	1.03
b	2070	1.25	0.423	0.414	1.02
			Average Col. 15		1.02
			Average Series III		0.985
			Standard Deviation		0.061

TABLE 3

TESTS OF CONCENTRICALLY LOADED LONG COLUMNS

Specimen	Concrete Strength f'_c, psi.	Cross-Section d x b, in. x in.	Slenderness ℓ/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
		Baumann 1930-1933 (14)				
I	2200	3.94 x 7.87	32.1	1.026	0.947	1.08
III	2330	5.51 x 5.51	22.9	1.285	1.221	1.05
V	3830	5.47 x 6.97	23.3	1.172	1.187	0.99
Va	3830	5.51 x 7.01	23.2	1.223	1.191	1.03
VI	3610	3.86 x 7.80	32.7	0.955	0.729	1.31
VIa	3610	3.94 x 7.87	32.1	0.950	0.748	1.27
VIII	4180	7.01 x 7.17	17.5	1.345	1.158	1.16
VIIIa	4180	7.09 x 7.09	15.5	1.531	1.182	1.30
3	4870	6.30 x 9.84	40.7	0.585	0.405	1.44
15	5630	6.34 x 9.72	40.4	0.418	0.386	1.08
			Average			1.171
			Standard Deviation			0.142
		Thomas, 1938 (17)				
LC1	3530	6.00 x 6.00	14.7	1.224	1.288	0.95
LC2	3840	6.00 x 6.00	20.7	1.043	1.125	0.93
			Average			0.940
		Hanson and Rosentröm, 1945-1946 (18)				
1a	5270	5.16 x 7.13	25.8	0.973	0.713	1.36
2a	5980	5.20 x 7.17	25.6	0.833	0.693	1.20
3a	7110	5.31 x 7.20	25.1	0.750	0.673	1.11
			Average			1.223
		Rambøll, 1949-1950 (19)				
13	4860	5.59 x 7.13	13.0	0.790	1.064	0.74
14	4380	5.59 x 7.13	13.0	1.040	1.080	0.96
15	4220	5.79 x 7.13	12.6	0.983	1.090	0.90
16	4220	5.75 x 7.20	12.7	0.980	1.089	0.90
27	5010	5.55 x 7.17	20.6	0.768	0.897	0.86
28	4880	5.75 x 7.20	19.9	0.642	0.905	0.71
33	4700	5.63 x 7.20	30.1	0.686	0.619	1.11
			Average			0.883
			Standard Deviation			0.125

TABLE 3 (Concluded)

Specimen	Concrete Strength f'_c, psi	Cross-Section d x b, in. x in.	Slenderness ℓ/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
Gehler and Hütter, 1940-1942 and 1951-1952 (20)						
Ia	3270	5.50 x 6.30	40	0.562	0.517	1.09
				0.601		1.16
Ib	3920	5.50 x 6.30	30	0.749	0.701	1.07
				0.776		1.11
Ic	3670	5.50 x 6.30	25	1.033	0.871	1.19
				1.108		1.27
Id	3560	5.50 x 6.30	20	1.042	1.027	1.01
				1.182		1.15
Ie	3560	5.50 x 6.30	15	1.276	1.076	1.19
				1.214		1.13
IIa	3520	5.50 x 6.30	40	0.703	0.672	1.05
				0.754		1.12
1	4250	5.50 x 6.30	40	0.312	0.422	0.74
				0.352		0.83
2	4430	5.50 x 6.30	40	0.376	0.524	0.72
				0.491		0.94
3	4100	5.50 x 6.30	40	0.606	0.794	0.76
				0.531		0.67
4	2690	5.50 x 6.30	30	0.744	0.816	0.91
				0.717		0.88
5	3790	5.50 x 6.30	30	0.626	0.710	0.88
				0.658		0.93
6	4980	5.50 x 6.30	30	0.621	0.640	0.97
				0.621		0.97
			Average			0.989
			Standard Deviation			0.163
Ernst, Hromadik and Riveland, 1952 (15)						
3	3430	6.00 x 6.00	15	1.046	1.179	0.89
4	3430	6.00 x 6.00	25	0.961	0.894	1.07
			Average			0.980

NOTE: Only specimens with ℓ/d larger than 10 are listed in this table.

TABLE 4

TESTS OF ECCENTRICALLY LOADED LONG COLUMNS

Specimen	Concrete Strength f'_c, psi.	Cross-Section d x b, in. x in.	Slenderness ℓ/d	Eccentricity Ratio e_2/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
		Baumann, 1930-1933 (14)					
Ia	2290	3.94 x 7.87	32.1	0.083	0.566	0.63	0.90
IIIa	2360	5.51 x 5.51	22.9	0.083	0.869	0.89	0.98
4	4670	9.84 x 9.84	12.0	0.166	0.562	0.63	0.89
5	4640	4.92 x 9.84	25.9	0.166	0.404	0.32	1.26
6	4670	6.30 x 9.84	40.7	0.166	0.206	0.15	1.37
7	3480	9.84 x 9.84	11.7	0.166	0.662	0.68	0.97
8	3480	4.96 x 9.84	25.6	0.166	0.366	0.36	1.02
9	3460	6.38 x 9.84	40.2	0.166	0.251	0.18	1.39
10	5100	9.88 x 9.96	11.7	0.333	0.364	0.41	0.89
11	5100	4.96 x 9.92	25.6	0.333	0.207	0.15	1.38
12	5070	6.38 x 9.84	40.2	0.333	0.094	0.09	1.04
13	5600	9.88 x 9.72	11.8	0.333	0.344	0.38	0.91
14	5600	4.96 x 9.76	25.6	0.333	0.159	0.14	1.14
19*	4070	5.11 x 9.84	24.7	0.166	0.501	0.51	0.98
26*	5070	9.84 x 9.92	12.6	0.166	0.707	0.70	1.01
32*	4680	9.84 x 9.84	12.6	0.166	0.786	0.79	0.99
					Average		1.070
					Standard Deviation		0.175
		Thomas, 1938 (17)					
LC3	3450	6.00 x 6.00	23.7	0.005	1.018	0.99	1.03
LC4	3440	6.00 x 6.00	26.7	0.005	0.994	0.89	1.12
LC8	3330	6.00 x 6.00	14.7	0.027	1.049	1.01	1.04
LC7	3810	6.00 x 6.00	20.7	0.037	0.894	0.88	1.02
LC6	3970	6.00 x 6.00	23.7	0.042	0.830	0.75	1.11
LC5	3640	6.00 x 6.00	26.7	0.047	0.921	0.74	1.24
LC12	3030	6.00 x 6.00	14.7	0.027	1.063	1.13	0.94
LC11	2700	6.00 x 6.00	20.7	0.037	1.139	1.01	1.13
LC10	2730	6.00 x 6.00	23.7	0.042	1.006	0.86	1.17
LC9R	2900	6.00 x 6.00	26.7	0.017	0.912	0.91	1.05
PLC1	2720	3.00 x 3.00	33.1	0.056	0.884	0.87	1.02
PLC2	2800	3.00 x 3.00	33.1	0.056	0.850	0.85	1.00
					Average		1.073
					Standard Deviation		0.080

TABLE 4 (Cont'd.)

Specimen	Concrete Strength f'_c, psi.	Cross-Section d x b, in. x in.	Slenderness ℓ/d	Eccentricity Ratio e_2/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
Hanson and Rosenström, 1945-1946 (18)							
1b	5270	5.16 x 7.13	25.8	0.166	0.417	0.30	1.39
2b	5980	5.20 x 7.13	25.6	0.166	0.356	0.28	1.27
3b	7110	5.31 x 7.20	25.1	0.166	0.349	0.26	1.34
				Average			1.333
Rambøll, 1949-1950 (19)							
17	4280	5.59 x 7.09	13.0	0.083	0.902	0.76	1.19
18	4030	5.67 x 7.13	12.8	0.083	0.866	0.77	1.12
19	4120	5.59 x 7.09	13.0	0.166	0.762	0.60	1.27
20	4150	5.63 x 7.17	12.9	0.166	0.805	0.60	1.34
21	3930	5.71 x 7.20	12.7	0.333	0.481	0.38	1.27
22	3970	5.67 x 7.17	12.8	0.333	0.502	0.38	1.32
23	4000	5.67 x 7.13	12.8	0.666	0.154	0.13	1.18
24	3750	5.67 x 7.13	12.8	0.666	0.164	0.14	1.17
25	4810	5.67 x 7.17	12.8	0.833	0.093	0.08	1.16
26	4560	5.55 x 7.13	13.1	0.833	0.098	0.08	1.16
29	5040	5.67 x 7.17	20.1	0.166	0.431	0.42	1.03
30	4630	5.63 x 7.17	20.3	0.333	0.278	0.24	1.16
31	4980	5.67 x 7.20	20.1	0.666	0.094	0.09	1.04
32	5040	5.59 x 7.20	20.4	0.833	0.074	0.07	1.06
34	5050	5.71 x 7.17	29.6	0.083	0.527	0.38	1.39
35	4540	5.67 x 7.20	29.9	0.166	0.336	0.33	1.02
36	4590	5.63 x 7.20	30.1	0.666	0.080	0.08	1.00
37	4650	5.71 x 7.17	29.6	0.333	0.164	0.15	1.09
38	5540	5.71 x 7.17	29.6	0.833	0.051	0.05	1.02
				Average			1.157
				Standard Deviation			0.115
Gehler and Hütter, 1940-1942 and 1951-1952 (20)							
7	3020	5.50 x 6.30	15	0.0376	1.196 1.134	1.11	1.08 1.02
8	4200	5.50 x 6.30	20	0.051	0.873 0.810	0.85	1.03 0.95
9	4110	5.50 x 6.30	30	0.075	0.597 0.555	0.56	1.07 0.99
10	3260	5.50 x 6.30	40	0.101	0.357 0.388	0.38	0.94 1.02
11	3210	5.50 x 6.30	15	0.075	1.029 1.117	0.96	1.07 1.16

TABLE 4 (Concluded)

Specimen	Concrete Strength f'c, psi.	Cross-Section d x b, in. x in.	Slenderness ℓ/d	Eccentricity Ratio e_2/d	$\frac{P_{test}}{f''_c bd}$	$\frac{P_{calc}}{f''_c bd}$	$\frac{P_{test}}{P_{calc}}$
12	4200	5.50 x 6.30	20	0.101	0.721 0.735	0.72	1.00 1.02
13	4390	5.50 x 6.30	30	0.150	0.411 0.418	0.42	0.98 1.00
14	2830	5.50 x 6.30	40	0.201	0.341 0.306	0.32	1.07 0.96
15	3210	5.50 x 6.30	15	0.113	0.957 1.043	0.87	1.10 1.20
16	3020	5.50 x 6.30	20	0.152	0.668 0.777	0.71	0.94 1.09
17	3710	5.50 x 6.30	30	0.225	0.403 0.358	0.38	1.06 0.94
18	2890	5.50 x 6.30	40	0.302	0.276 0.266	0.26	1.06 1.02
				Average			1.032
				Standard Deviation			0.066
		Ernst, Hromadik and Riveland, 1952 (15)					
7	3430	6.00 x 6.00	15	0.125	0.761	0.74	1.03
8	3430	6.00 x 6.00	25	0.250	0.618	0.50	1.24
11	3430	6.00 x 6.00	15	0.250	0.554	0.52	1.07
12	3430	6.00 x 6.00	25	0.250	0.368	0.33	1.12
13	3430	6.00 x 6.00	25	0.375	0.236	0.24	0.98
				Average			1.088
				Standard Deviation			0.089

*End eccentricity $e_1 = 0$; in all other columns $e_1 = e_2$.

TABLE 5

SUMMARY OF P_{test}/P_{calc} FOR LONG COLUMNS

Investigation	Type of Loading	No. of Columns	Test Results, $\frac{P_{test}}{P_{calc}}$	
			Average	Standard Deviation
Baumann (14)	C[x]	10	1.171	0.142
	E[xx]	16	1.070	0.175
Thomas (17)	C	2	0.965	-----
	E	12	1.073	0.080
Hanson and Rosenström (18)	C	3	1.123	-----
	E	3	1.333	-----
Ramb∅ll (19)	C	7	0.883	0.125
	E	19	1.157	0.118
Gehler and Hutter (20)	C	24	0.989	0.163
	E	24	1.032	0.066
Ernst, Hromadik and Riveland (15)	C	2	0.980	-----
	E	5	1.088	0.089

Concentrically Loaded Columns, 48 specimens	
Average P_{test}/P_{calc}	1.024
Standard Deviation	0.178
Eccentrically Loaded Columns, 79 specimens	
Average P_{test}/P_{calc}	1.091
Standard Deviation	0.129

[x]C = concentrically loaded columns

[xx]E = eccentrically loaded columns

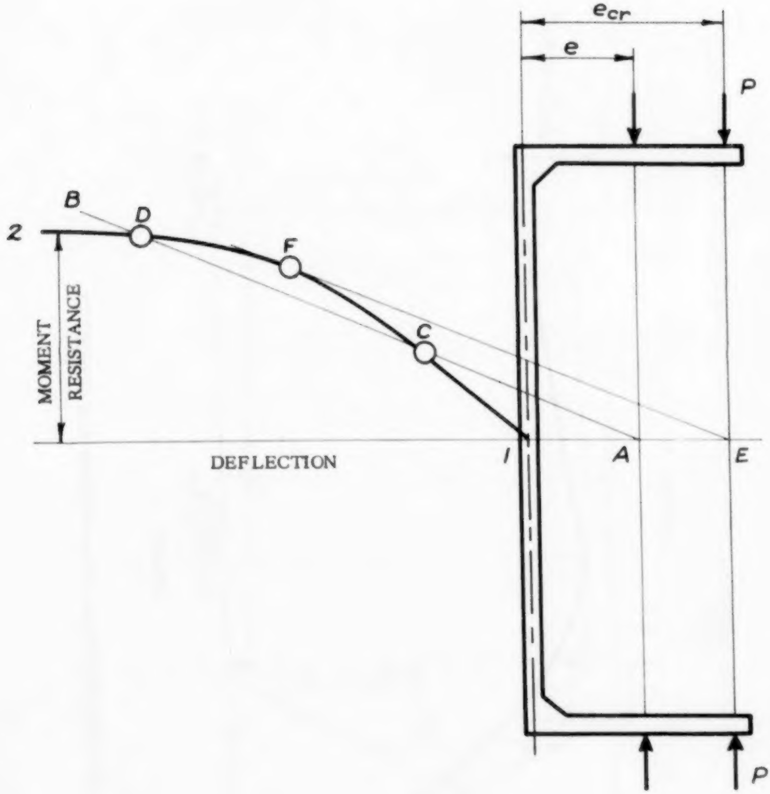

FIG. 1 BUCKLING OF A HINGED COLUMN

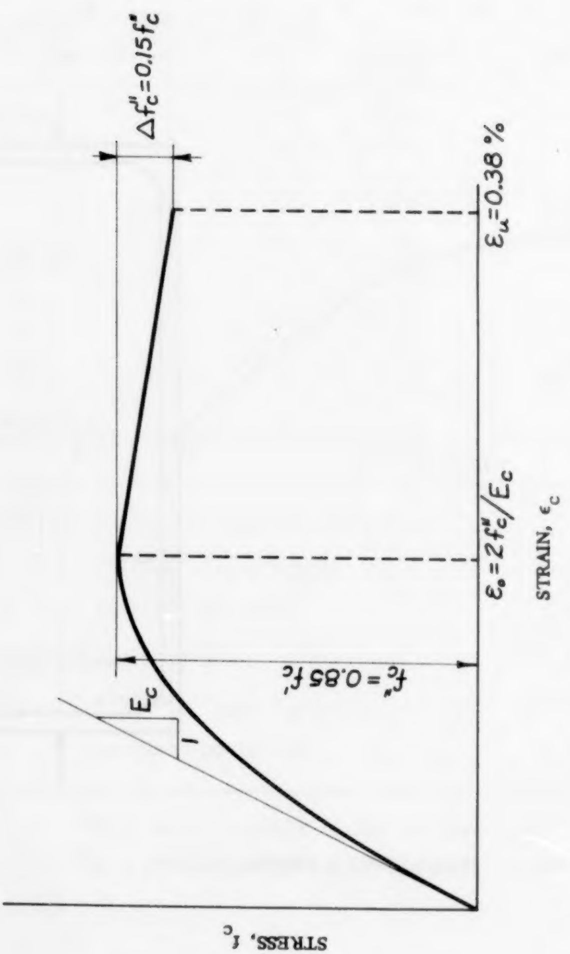

FIG. 2 ASSUMED STRESS-STRAIN RELATIONSHIP FOR CONCRETE

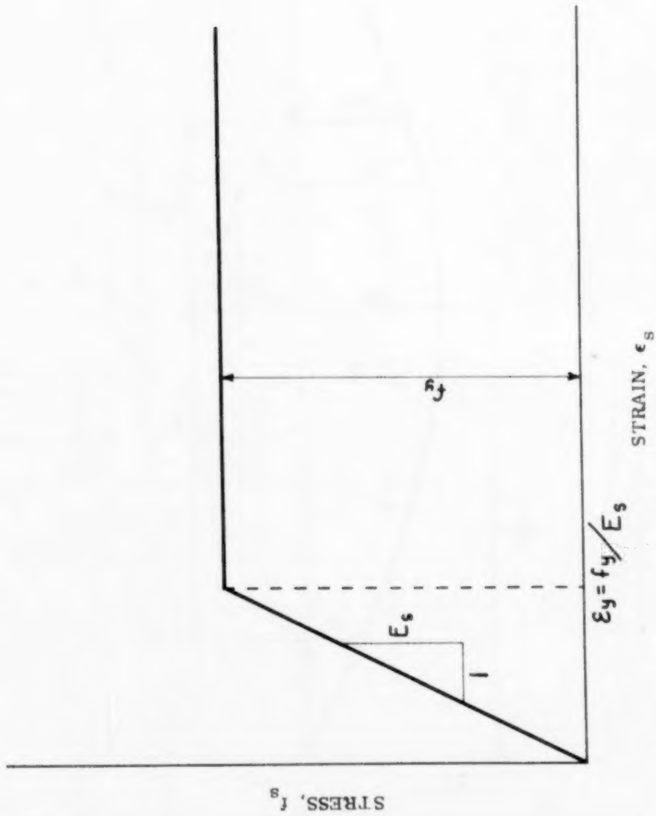

FIG. 3 ASSUMED STRESS-STRAIN RELATIONSHIP FOR REINFORCING STEEL

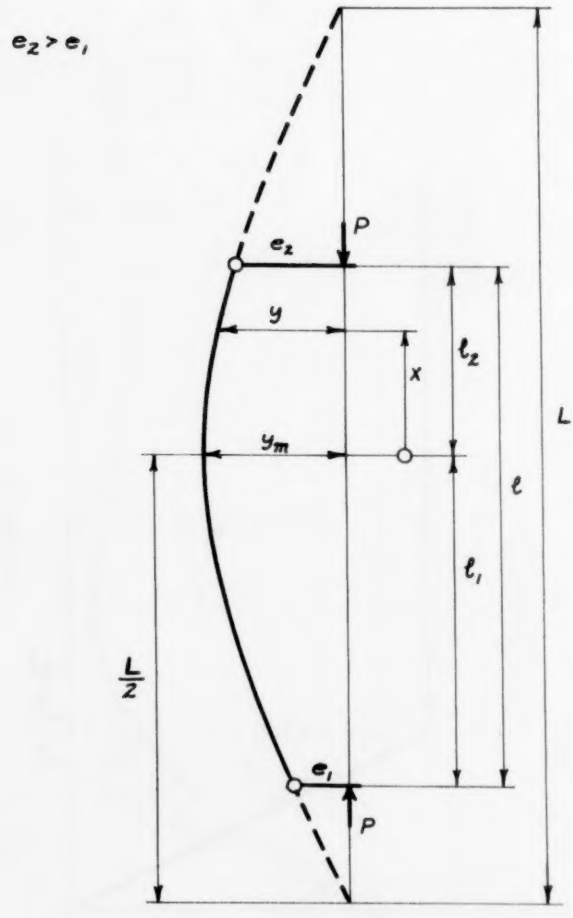

FIG. 4 DEFLECTED SHAPE OF AN ECCENTRICALLY LOADED HINGED COLUMN

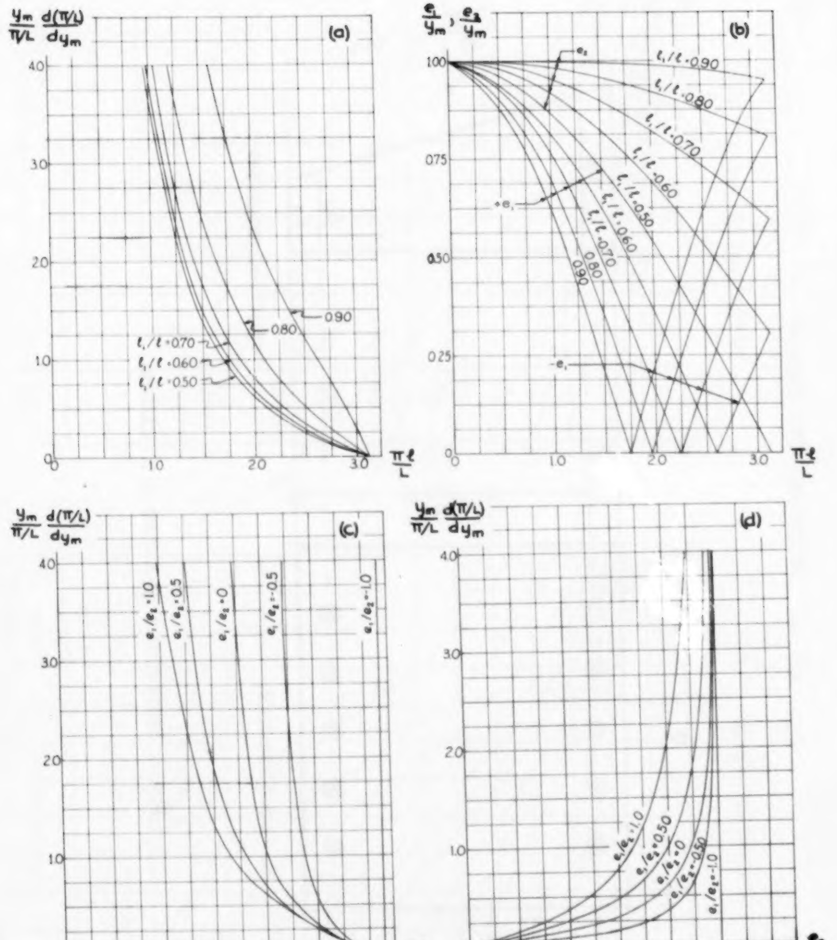

FIG. 5 SOLUTION OF EQ. 12

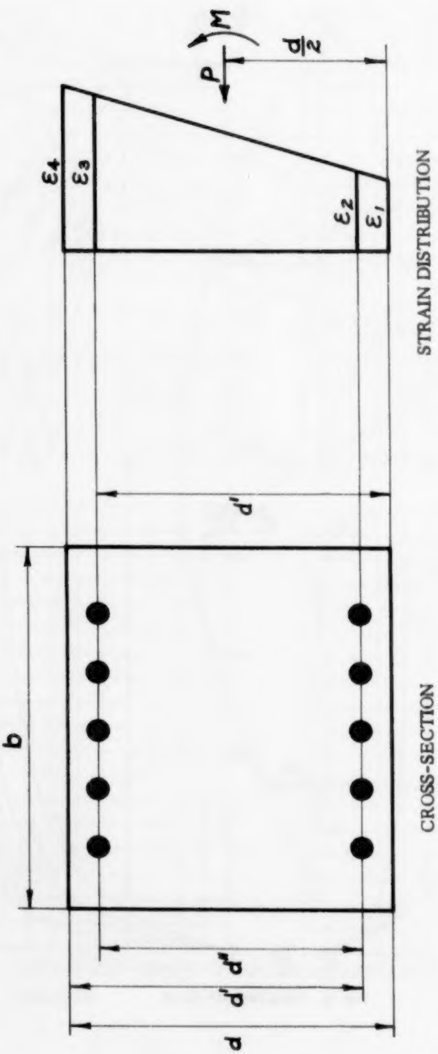

FIG. 6 DISTRIBUTION OF STRAIN IN A COLUMN

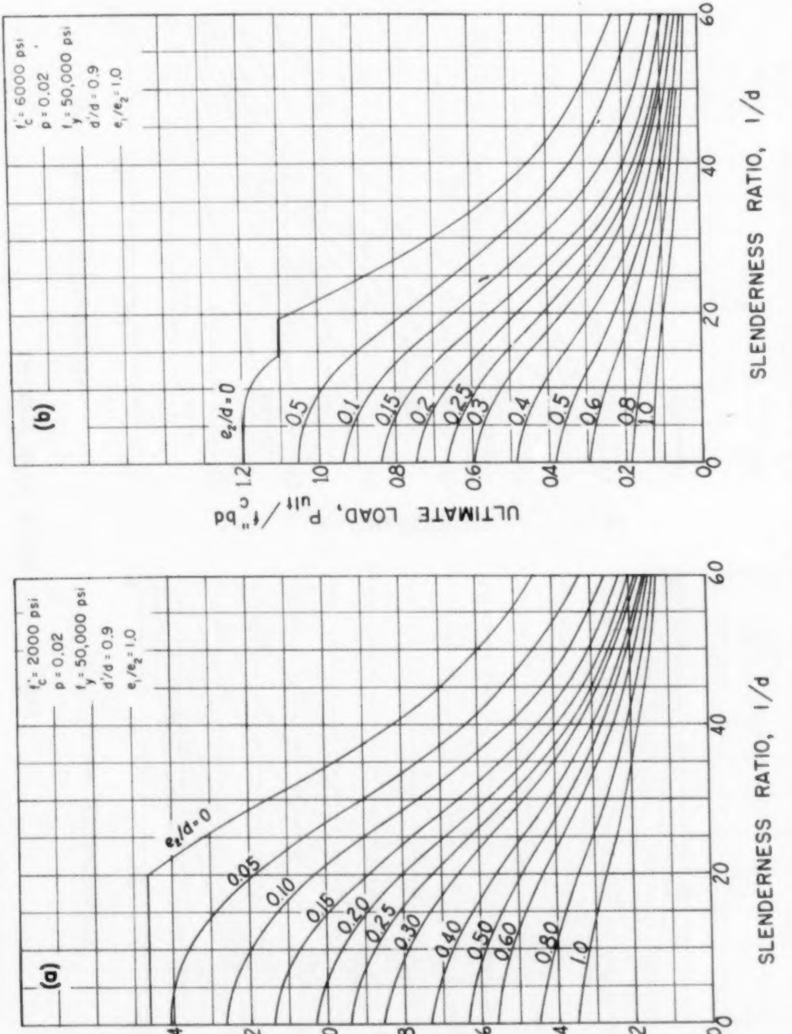

FIG. 7 ULTIMATE LOAD GRAPHS

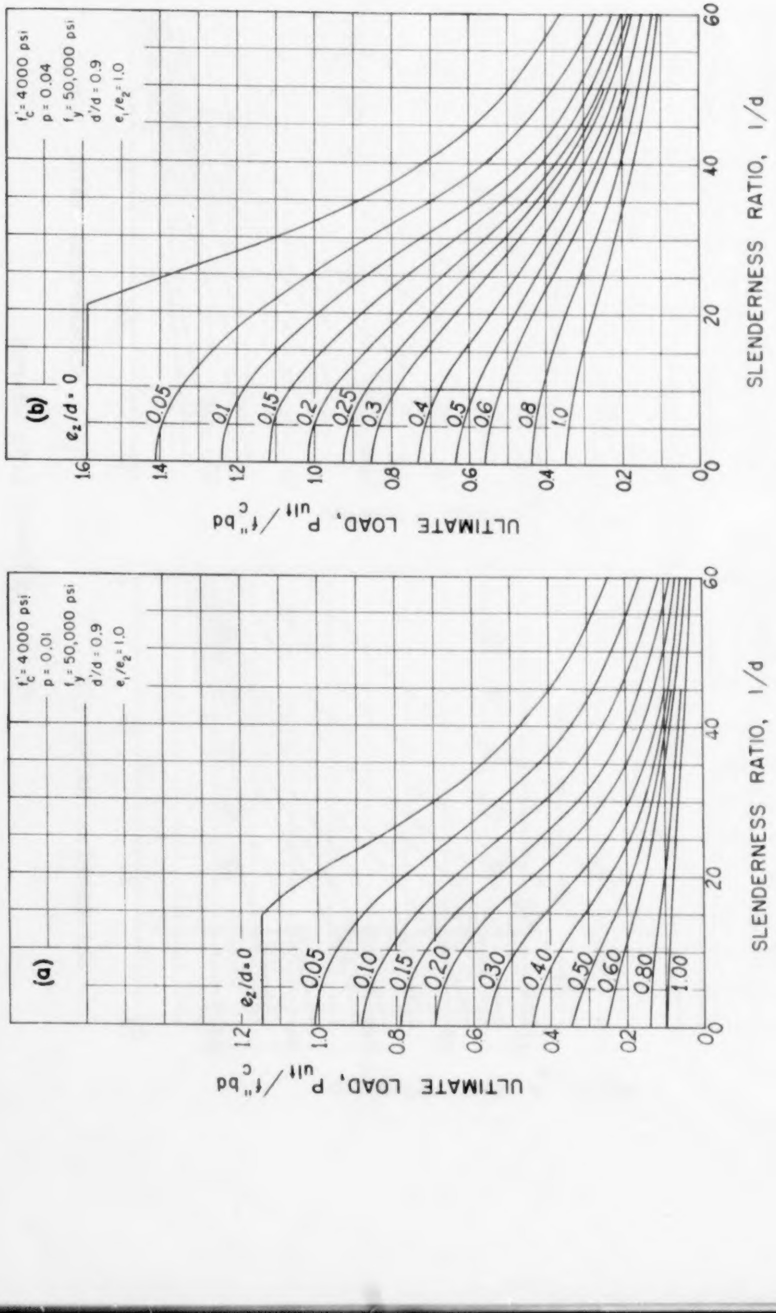

FIG. 8 ULTIMATE LOAD GRAPHS

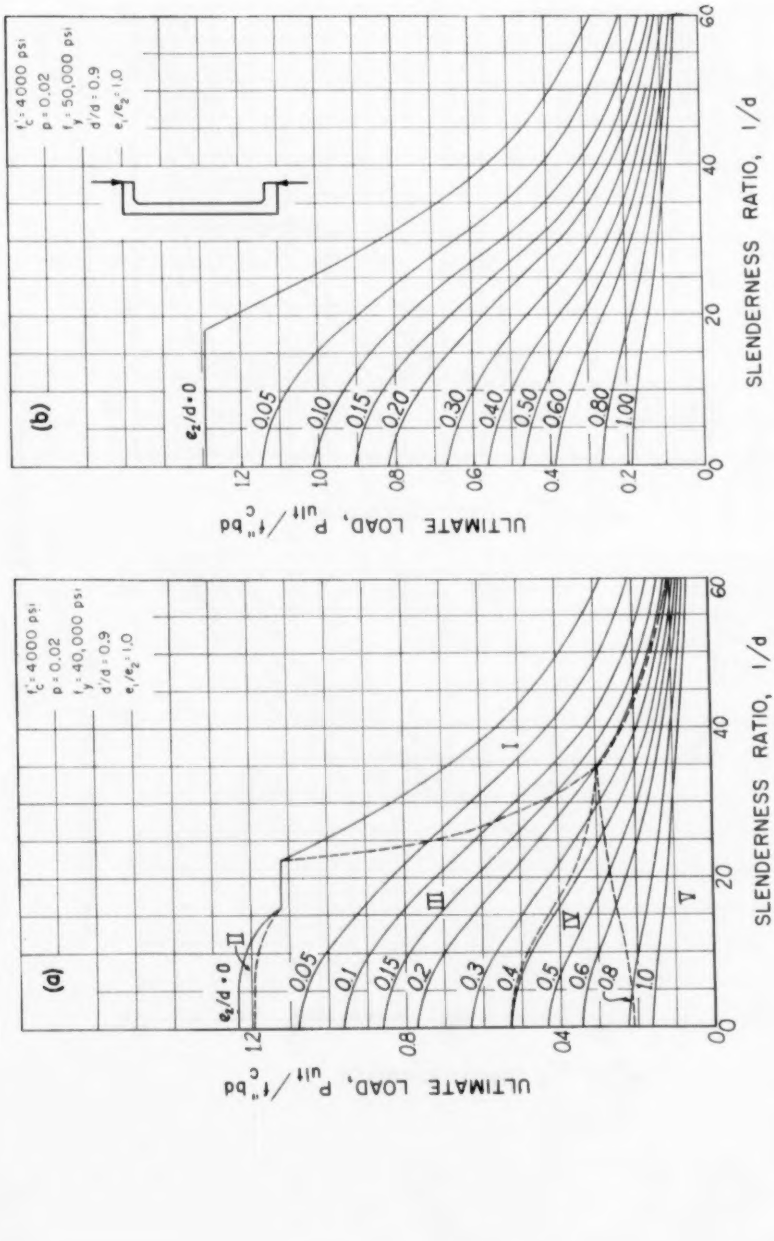

FIG. 9 ULTIMATE LOAD GRAPHS

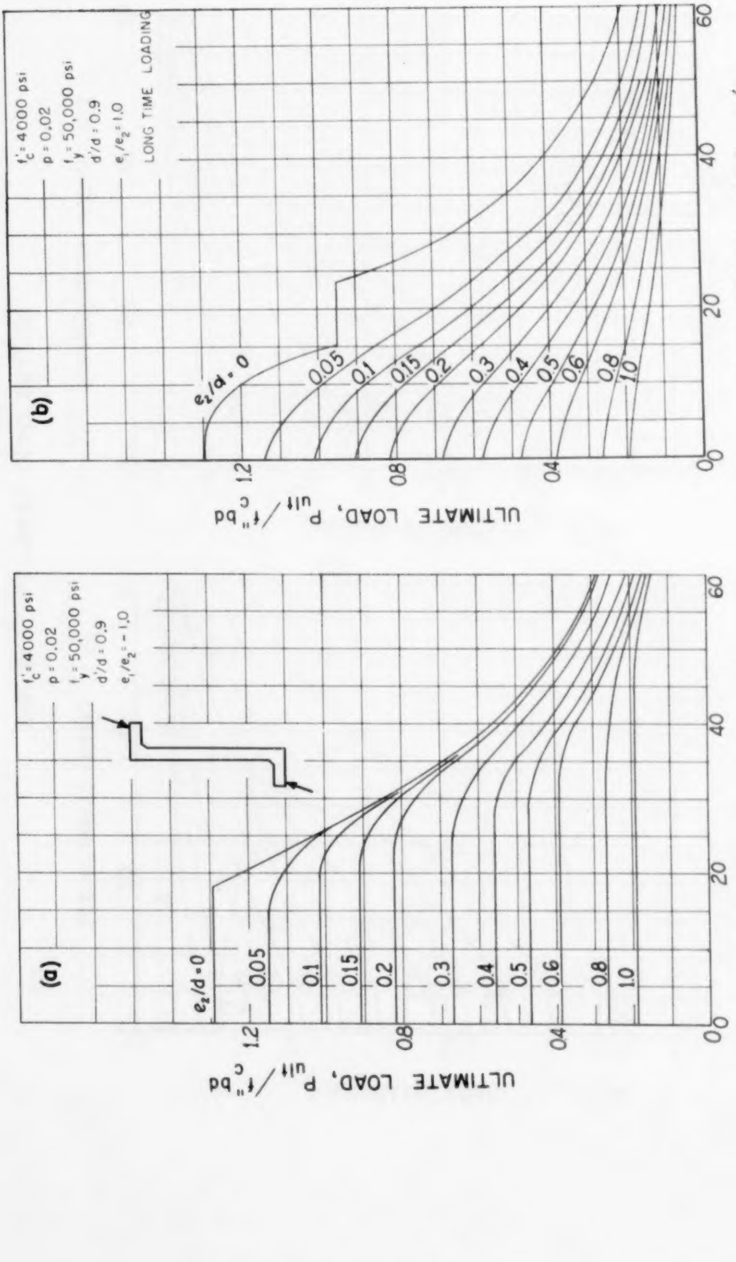

FIG. 10 ULTIMATE LOAD GRAPHS

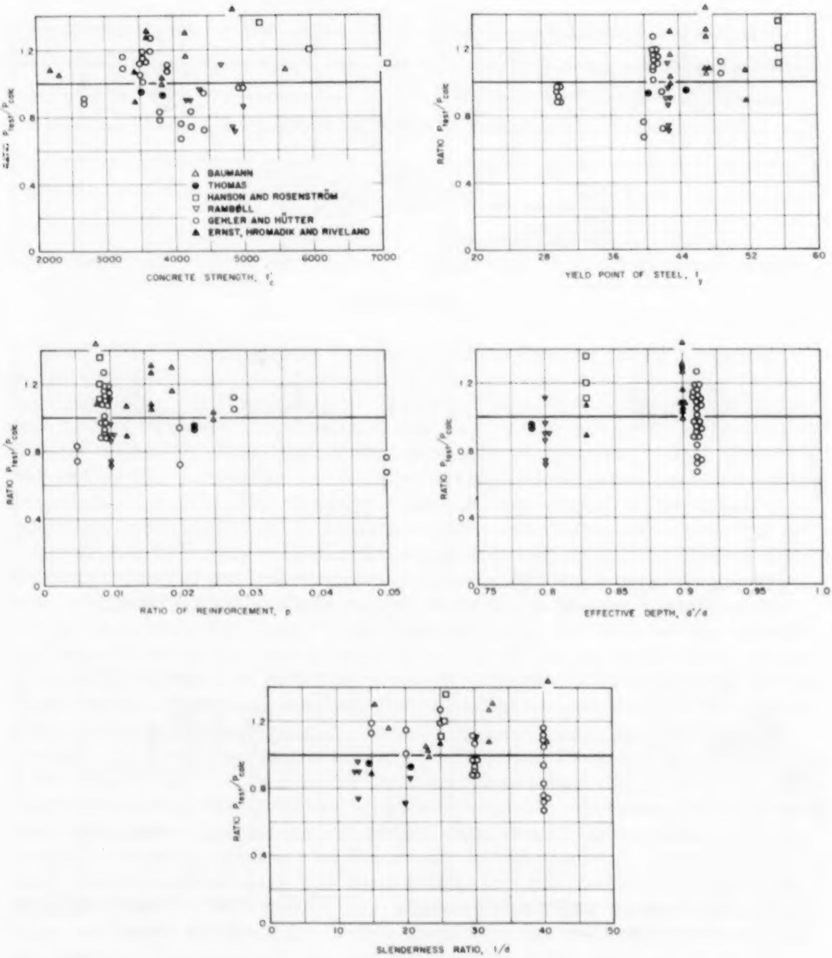

FIG. 11 TEST RESULTS CONCENTRICALLY LOADED COLUMNS

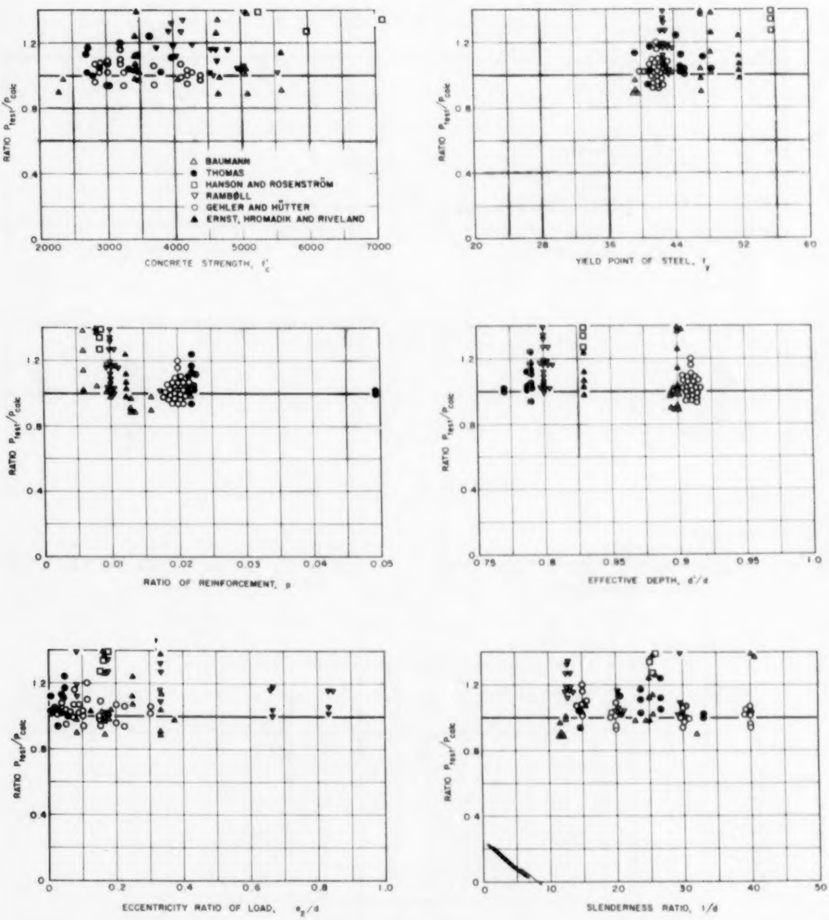

FIG. 12 TEST RESULTS ECCENTRICALLY LOADED COLUMNS

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THE STRUCTURAL PROPERTIES OF MAGNETITE CONCRETE[a]

Jerome M. Raphael,[1] M. ASCE
(Proc. Paper 1511)

SUMMARY

Magnetite concrete for the Engineering Test Reactor (ETR) biological shield was made from a combination of crushed magnetite aggregate and a low-alkali Type I portland cement. Some combinations of cement and aggregate used in concrete are potentially self-destructive through internal volume changes caused by alkali-aggregate reaction or by thermal growth. Tests proved that this particular combination was non-reactive and without deleterious thermal growth. The elongated shape and high density of the crushed magnetite particles tended to cause harshness and segregation in mixing the concrete, but by paying very careful attention to the proportion of fines and the composition of the cement paste in the mix, workable concrete mixes were designed having densities ranging from 225 to 239 psf. It proved important to prevent the cement paste from becoming too fluid, as this caused excessive bleeding and segregation. It was found that up to 4% of the weight of the mix could be lost by drying of small cylinders of the concrete, but this moisture loss could be completely controlled by encasing the shield in steel plate. Concrete of any required strength could be made, but for concretes having compressive strengths less than 4000 psi at 28 days, extra fines had to be added to the mix in place of cement to lend supporting power and lubricity to the paste. Elasticity and Poisson's ratio for this dense concrete were about 50% higher than for normal weight concretes of comparable compressive strength, varying with the density of the concrete. At the end of a year, the drying shrinkage was about 0.06%, and the coefficient of thermal expansion was 5×10^{-6} in/in/1° F. When tested against narrow and wide beam radiation, the mass absorption coefficient for this magnetite concrete was found to be 0.054 cm^2/gm, and the build-up factor was 2.57 at an absorbent thickness of 80 gm/cm^2.

Note: Discussion open until June 1, 1958. A postponement of this closing date can be obtained by writing to the ASCE Manager of Technical Publications. Paper 1511 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. ST 1, January, 1958.

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INTRODUCTION

The Engineering Test Reactor (ETR),[1] located at Arco, Idaho, was patterned on the Materials Test Reactor (MTR) which is also located in this same general area. The primary purpose of ETR is to provide in-core experimental facilities primarily for the aircraft nuclear propulsion program, as well as for general experimentation within the core of a reactor. The reactor uses flat plate enriched uranium-aluminum alloy fuel elements cooled and moderated by pressurized normal water, and is designed to operate at 175 megawatts.

During the initial design stages of ETR, when rather high operating temperatures were contemplated in the reactor, question arose as to the maximum temperature to which the concrete could be subjected. In studying the effects of high operating temperature in producing stress in the reactor shield, it became apparent how little information was available about the structural properties of the magnetite concrete which was under consideration for the shield, and it was determined that these properties should be established by direct test. At the same time, it became apparent that the environmental conditions being considered were similar to those under which large volume changes had taken place in some concretes, resulting in severe cracking, and investigation was made to be sure that the concrete combination contemplated for ETR would undergo no deleterious volume changes.

Maximum Concrete Temperature

In designing ETR, concern was expressed whether the properties of the biological shield concrete would limit the maximum operating temperature of the reactor. Although the maximum operating temperature had not yet been fixed, it was felt desirable to exceed the 50 degree differential temperature which had been cited in the literature.[2] Accordingly, an investigation was made to determine the physical basis for limiting the temperature of the inner-face of the concrete shield.

Operation of the reactor produces two sources of heat which affect the biological shield. In cooling the fuel elements in the reactor, the water is itself heated. Some of this heat flows out through the shell and thence through the concrete to the outside surface of the shield. The other source is the development of heat in the concrete of the shield itself as the radiation is attenuated by the shield. Acting together, these two sources produce a temperature gradient which is high near the inner surface of the shield, and at room temperature at the outer surface of the shield. The high temperatures and the differential temperatures affect the concrete of the shield in four ways.

1. High temperature may alter the internal structure of the concrete and affect its properties generally.
2. The range from high temperature inside to low temperature near the outer surface tends to cause a restrained differential expansion which stresses the shield structurally.
3. High temperatures and moist conditions of storage are necessary conditions for continuing volume changes caused by alkali-aggregate reaction if a deleterious combination of materials is already present in the concrete.

4. Cycles of temperature may cause increase in volume in the shield with consequent danger of wide cracking.

If concrete is allowed to become thoroughly dry before being subjected to high temperatures, it has been reported by many observers to be able to withstand temperatures of the order of 500° F(3, 4, 5, 6, 7) with no appreciable loss in strength. However, if concrete that is thoroughly saturated is suddenly subjected to high temperatures, the entrapped water will flash to steam, and if the steam pressure exceeds the tensile strength of the concrete, the concrete will spall violently. The vapor pressure of confined water at various temperatures is shown in the following table.

Table I - Pressure-Temperature Relations for Confined Water

Temperature °F	Gage Pressure, psi
212	0
300	53
400	235
422	300
500	665
600	1550

It was assumed that the concrete of the biological shield would have an ultimate compressive strength of 3000 psi, and a tensile strength of around 300 psi. It is thus apparent that the vapor pressure of the steam will equal the assumed tensile strength of the concrete at 422° F.

With a steady-state temperature differential in the biological shield from hot inside to cool outside, the inner concrete will tend to expand and will be restrained by the relatively cooler outer concrete, with the result that the inner concrete will be compressed and the outer concrete will develop tension. The greater the differential temperature, the greater the exterior tension. Some reactor designers have felt that the limit of differential temperature should be controlled by the cracking strength of concrete. However, for the design of ETR it was felt that by using normal steel reinforcing techniques, a greater temperature differential could be tolerated without producing undue cracking. Using criteria developed originally for the design of reinforced concrete chimneys,(8) a percentage of steel was found at which concrete and steel were operating at their maximum efficiency. For this percentage of steel, the differential temperature was found to be 165° F, which set the maximum inner temperature of the concrete shield at 230° F.

Aggregate

In setting the specifications for the shielding concrete for ETR, conventionally mixed concrete in the density range from 220 to 240 pcf was considered.

This can be secured with aggregates having a bulk specific gravity of about 4.6. The aggregates which have had much use in reactor shields and which have the required density are barite, and the iron ores limonite and magnetite. Of these three, magnetite seems to suffer the least from grinding in the mixer. Considering the economics of various heavy aggregate sources, and the type of application, magnetite from Lovelock, Nevada, was tentatively chosen as the aggregate for ETR, contingent on its successful performance in volume change tests. The magnetite ore supplied comprised about 81% magnetite, with the remainder consisting of a gangue of quartz, feldspar, chlorites, and ferro-magnesiums. Since the lighter materials of this ore break more readily than the magnetite, there is some variation of density with size, the smaller fractions of the aggregate being about 2% lighter than the coarser fractions.

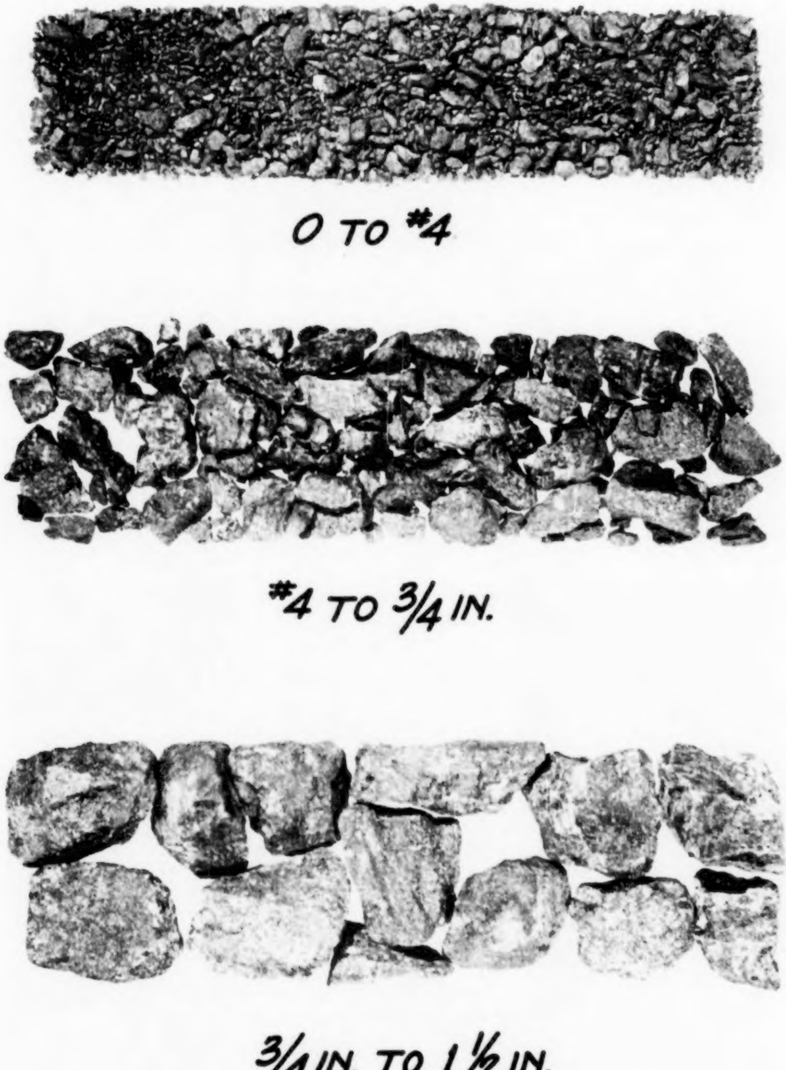
The shape of the particles as crushed is not particularly suited to making high quality workable concrete. The particles are angular and elongated, and the characteristic shapes shown in Fig. 1 can be seen in all sizes right down to the finest. This is far from the rounded or blocky shapes that make the most workable concrete. It was found by test that magnetite aggregate can absorb considerable quantities of water. In the short time that concrete is being mixed it can absorb about 1/4% of its weight of water, while with 24 hours of soaking, the magnetite can absorb up to 2% moisture. The aggregate was supplied in three sizes: sand up to 1/4" maximum; medium gravel from 1/4" to 3/4"; and coarse gravel from 3/4" to 1-1/2". Specifications for the gradation of the aggregates followed closely the ASTM Specifications for Concrete Aggregates, C33.

Cement

The cement used on ETR was a low-alkali Type I Cement manufactured in Idaho. The chemical and physical characteristics of this cement are shown in Table II. Examination of the table shows that the cement, while sold as type I, can also meet the specifications for type II because of its compound composition. This cement developed much less heat while setting than the normal type I cement, a distinct advantage when pouring large masses of concrete. In addition, the relatively slow initial set made it easier to consolidate large masses of concrete, at the expense of added strain on the forms. This is a low-alkali cement because the combined percentages of soda and potassa in terms of Na_2o are less than 0.60% by weight of the cement, which has been set as the practical maximum to prevent alkali-aggregate deterioration.

Effect of Volume Changes

If a concrete mass were to expand uniformly or contract uniformly due to some change of state, and if there were no restraint to such volume change, no stress and no cracking would result. If however, some agency were active in the concrete such that the interior of the mass expanded at a greater rate than the exterior, the interior of the mass would be compressed and the exterior would be stressed in tension, until, if the volume changes were sufficiently great, the exterior would crack. Fig. 2 shows the result of just such a reaction. This is the entrance pylon to a western dam, which, as can be



CRUSHED MAGNETITE AGGREGATE

FIG. 1

TABLE II

TYPE I. LOW-ALKALI PORTLAND CEMENT

CHEMICAL ANALYSIS

PERCENTAGE BY WEIGHT

Insol	0.11
SiO_2	23.40
Fe_2O_3	2.53
Al_2O_3	3.71
CaO	64.31
MgO	2.09
SO_3	1.96
Loss on Ignition	2.01

Na_2O	0.09
K_2O	0.68
$Na_2O + .658\ K_2O$	0.54

Computed Compound Composition

C_4AF	7.7
C_3A	5.6
C_3S	49.8
C_2S	29.6

PHYSICAL TESTS

Setting Time:
- Initial (Gilmore) 3 hrs. 5 min.
- Final (Gilmore) 5 hrs. 10 min.
- Initial (Vicat) 1 hrs. 50 min.

Autoclave Expansion........0.05%

Fineness
- Wagner 1690 cm^2/gm
- Blaine 2860 cm^2/gm

Air Content.........2.2%

Briquets:
- 3-day...........295 psi
- 7-day...........365 psi

Cubes:
- 3-day......... 2040 psi
- 7-day......... 2990 psi

FIG. 2 — CRACKING CAUSED BY ALKALI-AGGREGATE REACTION

seen, has developed cracks of the order of 1-1/2" wide, entirely unrelated to structural stresses, but which are the result of differential volume changes. The agency that is active in this particular case is alkali-aggregate reaction. A similar action occurs when the active agency is cyclic thermal growth.

While the aggregate in concrete is ordinarily considered inert, certain highly silicious constituents of the aggregate can react with the alkalis of the cement to produce new compounds which have a greater volume that the original constituents. This is called the alkali-aggregate reaction.(9, 10, 11) Two standard tests have been developed by ASTM to determine the potential reactivity of concrete materials. ASTM Method C289, based on the amount of chemical reaction of the aggregate with a sodium hydroxide solution under controlled test condition, determines the potential reactivity of the aggregate alone. ASTM Method C227, based on direct measurement of volume change, is used to determine the potential alkali reactivity of cement-aggregate combinations in mortar bars. For this test, 1" x 1" x 11" mortar bars are stored at 100° F over but not in contact with water in a sealed container, and the lengths of these bars are measured periodically. The combination is deemed to be potentially reactive if the volume changes exceed predetermined limits, as shown in Fig. 3. When this test was applied to the combination of aggregate and cement contemplated for ETR, the results were as shown in the figure. The magnetite aggregate low-alkali type I cement combination selected for ETR is shown to be non-reactive. However, a barite-aggregate-high-alkali type I cement combination contemplated for another application showed potential reactivity in a similar test. Also shown on this same figure is the behavior under test of Oro Fino sand with a high-alkali cement. This combination developed wide cracking in structures in which it was used. It might be well, therefore, to run similar tests on any unproved combination of materials contemplated for use in a reactor to assure that potentially dangerous combinations are not chosen.

In several instances, growth of concrete has been found to be directly related to cyclic temperature changes. The actual mechanism is believed to be an increased volume due to an interaction between magnesium oxides of the cement and some constituents of the aggregate. No standard test has yet been developed for thermal growth. However, for evaluating the potential behavior of the concrete contemplated for ETR, a test was made up patterned closely after ASTM Method C227. Mortar bars were made up as before and cured at 70 degrees in fog for 14 days. The bars were then water-proofed with neoprene rubber, and subjected to temperature cycles which consisted of moist storage for two days at 200 degrees followed by one day at 70° F. Lengths of the bars were measured each time the bars were cooled. The results of this test are shown in Fig. 4. All bars expanded 0.01% during the first heating cycle, an amount which corresponds to a 20° F temperature rise. The expansion of 0.01% on the first heating can be compared with the reported autoclave expansion of 0.05% for the cement alone. Considering the more complete state of hydration of the mortar bars, the dilution of the paste with the aggregate, and the lower test temperature compared with the standard autoclave test temperature, the expansion of 0.01% on the first cycle observed for the three cement-aggregate combinations shown seems reasonable. Following the initial heating, no further thermal growth was shown by any of the combinations tested. For purpose of comparison, the results of experience with concrete at Conowingo power plant are also shown on this figure. This concrete has gradually expanded under the influence of temper-

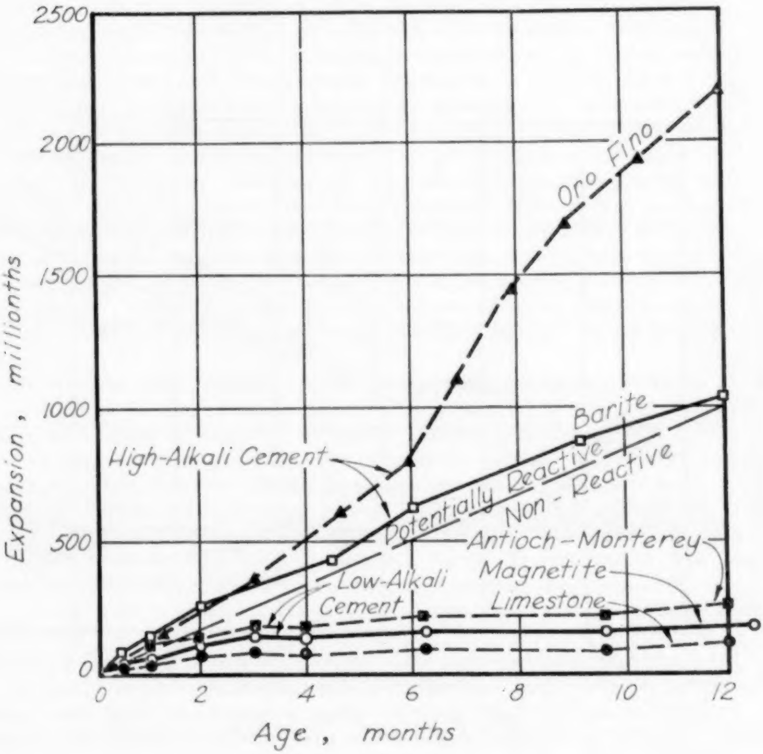


Fig. 3 - Potential Alkali Reactivity

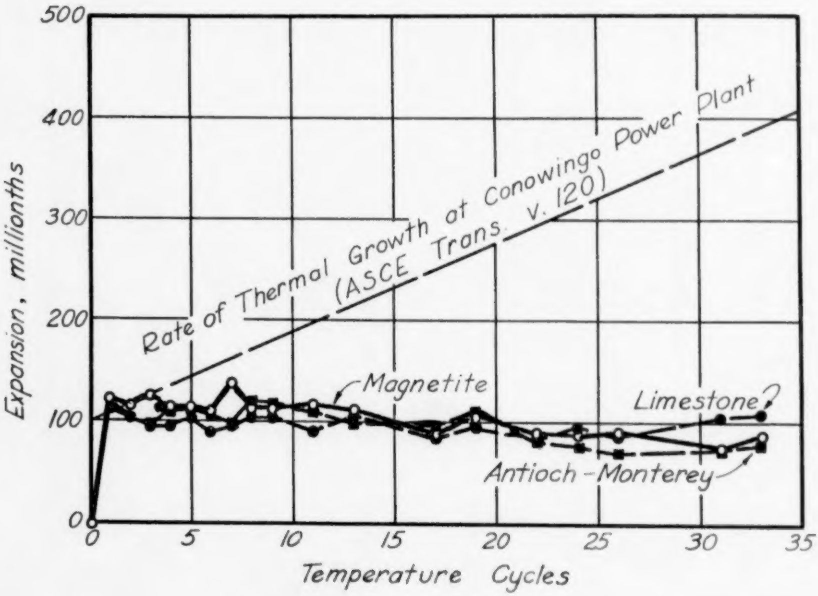


Fig. 4 - Cyclical Thermal Growth

ature cycles, causing binding of control valves in the power plant and other evidence of structural distress.

Concrete Mix Design

The concrete mix for a reactor shield works best when it has maximum density for gamma radiation shielding and when it contains as much hydrogen as possible, usually in the form of water, for neutron shielding. If there were no other requirements that must be met simultaneously, these could easily be met with a mix based on minimum sand and the maximum amount of large aggregate, with high water content and minimum cement. Such a mix would certainly segregate badly and bleed excessively, but this paradoxically would make the concrete even better for these simple requirements, since the large aggregates would pack down into a tighter mass as the paste bled out to the top of the mix. However, for ETR the requirements are not quite this simple. The operating temperature differential through the shield induces circumferential and longitudinal stresses which necessitate a certain amount of strength. Various embedded items in the shield require that segregation be held to a minimum to prevent the formation of gaps in the shielding underneath the embedded items. The concrete must be workable in order that it can be moved easily into intricate passages and around the embedded items.

Some of these requirements proved mutually incompatible. Reduced segregation and increased workability were accomplished simultaneously at the expense of reduced density. At first it seemed like a relatively easy matter to vary the strength by simply varying the cement content. Trial mixes quickly proved, however, that the cement has a dual function in heavy concrete mixes: it not only is the strength element in the hardened concrete, but it is needed in the fresh concrete to float the heavy aggregate particles and to provide workability and cohesion to the mix. In addition, the flat, angular, and elongated shape of all particles, right down to the finest sand, required a mix able to lubricate all particles more than usual for normal workability.

In all, fifteen trial mixes were made. The first mix was based on proportions which had proved successful on another job using other materials. Modifications were made on successive mixes to improve one or another property. Mix proportions and characteristics are shown in Table III. While the proportions are shown for this particular combination of cement and aggregate, it must be borne in mind that these results may be applicable to this particular concrete only, as far as details of proportions are concerned, and that for other ingredients, other mix proportions will be probably necessary.

However, from all these trials, several guiding principles may be derived for proportioning and adjusting concrete mixes. The paste must be carefully proportioned so that it is tough and dense enough to float the aggregate particles, otherwise it will run out of the mix and the concrete will have excessive segregation. Water cannot be added indiscriminately to increase slump, as the heavy concrete will then segregate rather than slump as a cohesive mass. Thus, in order to make a workable heavy concrete, the large particles must be floated in a dense, tough mortar. This eventually meant minimum water consistent with the required slump and a maximum amount of fine particles either in the form of cement, air, or inert admixtures. Since air reduces

TABLE III - MAGNETITE CONCRETE MIX STUDIES

Mix No.	Cement Content Sacks/cu yd	Sand Content %*	Water Content gal/cu yd	W/C Ratio by wgt.	Density lb/cu ft	% Air by Vol.	Slump In.	Bleeding %	Batch Weights in lbs. for 1 cu yd-(Saturated Surface Dry) Cement	Water	Sand	Medium Agg.	Coarse Agg.	Admixtures	Strength psi 6"x12" cyls 7 Day	28 Day	90 Day	Remarks
1.	5.9	34	40	0.60	234	0.9	0.5	4.1	556	334	1850	2130	1430		-	-	-	Under-sanded, harsh, low cohesion
2.	5.9	34	40	0.60	231	0.9	1.1	6.1	556	334	1850	2130	1430	Densifier	-	-	-	Workable, bled excessively
3.	5.9	34	42	0.63	232	0.6	2.8	4.0	553	350	1820	2100	1440		2600	-	-	Under-sanded, segregated badly, fell apart
4.	5.8	45	39.5	0.61	228	1.3	1.4	2.9	542	330	2360	1720	1210		2780	-	-	Sand o.k., cohesive, fatty
5.	4.9	40	39	0.70	233	1.3	1.3	2.6	463	324	2200	1290	1990		2180	-	-	Sand o.k., excessive large rocks, req'd vibration
6.	4.9	40	40.5	0.73	233	1.1	1.5	3.5	463	337	2200	1640	1640		2020	-	-	Sand o.k., cohesive, good workability
7.	5.0	37	42	0.75	236	0.8	2.0	2.9	468	352	2070	1750	1750		1890	-	-	Under-sanded, non-cohesive, fell apart
8.	4.7	40	42.5	0.80	234	0.6	5.5	5.8	437	350	2220	1660	1660		1460	2170	-	Soupy, fell apart, segregated badly
9.	4.7	40	39	0.73	236	0.7	3.5	3.2	443	322	2240	1680	1680		1820	2790	3430	Same as 8 with less water, segregated, non-cohesive, used for "A" series
10.	5.0	44.5	34.5	0.61			0		470	287	2540	1810	1350					Too dry to mold, no slump, harsh, non-plastic
11.	5.0	44.5	38	0.68	239	1.2	0.5	1.8	466	317	2520	1800	1340		-	3380	-	Add water to 10; cohesive, not harsh, vibration req'd
12.	4.8	44.5	37	0.68	233	2.6	0.8	3.1	454	310	2460	1750	1310	Air Entrainment	-	3300	-	Add air to 11: Seemed to lack fines
13.	5.0	46	38	0.68	233	2.4	1.3	1.8	470	318	2540	1700	1270	"	-	3440	-	Add -100 sand to 12: Better but still needs fines
14.	5.8	47	40	0.61	225	3.6	3.5	3.8	546	331	2440	1660	1090	"	-	3930	-	Add cement to 13: Cohesive, workable, fatty, non-segregating
15.	4.8	47	42	0.67**	225	3.2	3.3	3.0	453	348	2440	1660	1090	" plus 68 lb pozzolan	1850	3110	4010	Replace 1 sack cement with equal vol. pozzolan, workable, fat, stickier than 14, "B" series

* Percent of sand by wgt. of total aggregate

**Water/(Cement + Pozzolan)

density and cement increases strength, when a low strength mix is desired, a pozzolanic admixture may be needed to obtain optimum density, workability, and strength.

To some extent, the final mix to be used in the field will be dependent on the actual shape of aggregate grains produced by the aggregate supplier. If the grains are rough, angular, flat and elongated like those produced by the laboratory crusher used in the trials, then some rounded fines will be necessary to lubricate the mix. If, on the other hand, the particles are blocky or subrounded, it may be possible to cut down on the sand content and the amount of fines in the sand. Air acted as a fine material in increasing workability, but it also reduced density. Its use will depend on how critical is the requirement for density.

Density

The densities of the fifteen trial mixes ranged from 225 pcf to 239 pcf. Maximum density of 239 pcf was found in Mix 11, which seemed low on fines, had very little slump, and required vibration for compaction. This mix could be used in the immediate neighborhood of the reactor core, if placing conditions were not too difficult. When this mix was modified for greater workability, the lightest mix of 225 pcf was produced. This might be quite suitable for parts of the shield where the intensity of radiation is somewhat reduced and where many embedded items make placement of concrete difficult.

Loss of Weight on Drying

Field tests showed that a loss of weight of as much as 5% was possible in completely dried out heavy concrete specimens. Since the water in the mix is only about 5% of the total weight of the ingredients, and since some of the water must combine chemically with the cement to produce new stable compounds, it was at first presumed impossible to obtain that much weight loss. However, tests in the laboratory of 3" x 6" cylinders dried for a week at 220° F showed a weight loss of just over 4%. This is evidently composed of all uncombined water in the paste plus all absorbed water in the magnetite. In a second test two 3" x 6" cylinders were dried continuously at 130° F, which approximates the operating temperature of the inner face of the concrete shield. The results, plotted in Fig. 5, show that the same weight loss of 4.2% was reached in 35 days, or 5 times the period needed at the higher temperatures. In the 3" x 6" cylinder, moisture moves in three dimensions to all surfaces. If the 8-ft. thick concrete shield were left bare on the outer and inner face, moisture would move along one direction, radially. The similarity of the general laws governing moisture and heat losses(13) was used as a guide to extrapolate the weight loss of the small cylinder to an equal condition of weight loss in the 8-ft. thick shield. Computations showed that under similar temperature conditions, it would take over 80 years for the shield to lose 4% of its density by moisture loss. However, since the reactor shield is faced outside and inside with plate steel, evaporation will be effectively impeded, and the problem of weight loss through evaporation seems pointless.

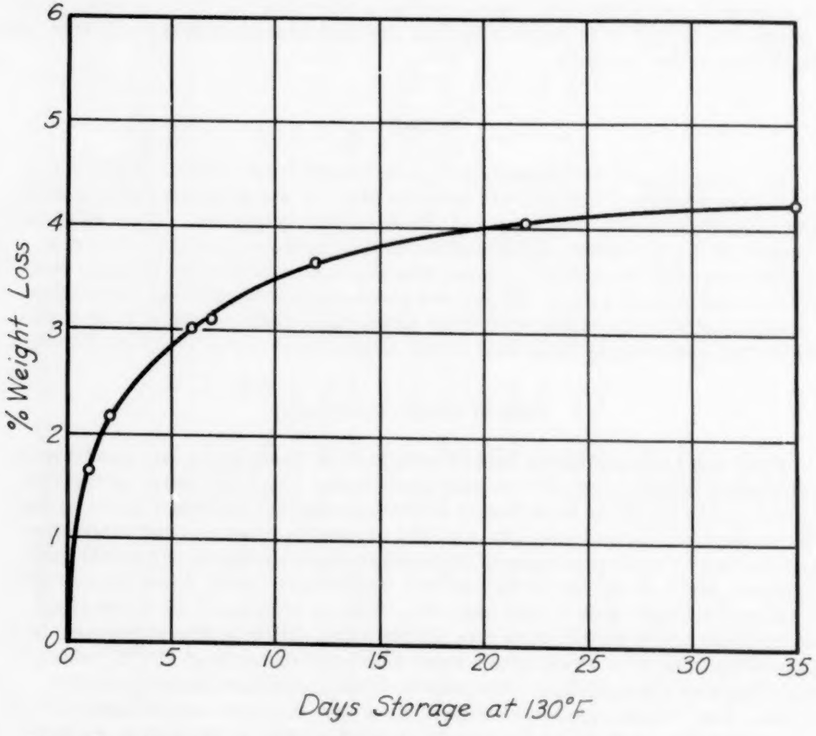


Fig. 5 - Weight Loss of 3 by 6-in Concrete Cylinder

Bleeding

High density concretes, having large discrepancies between the density of aggregate and cement paste, are very sensitive to minor changes in the composition of the paste. Unless the paste is made dense and tough, the relatively heavier large aggregate particles will settle in the paste, and force the paste to float upward, producing excessive bleeding, and what is almost synonymous, excessive segregation. It can be readily visualized what this would mean to the effectiveness of the reactor shield. At many locations, horizontal pipes of large diameter traverse the shield. If there were excessive bleeding, concrete would actually settle away from the pipes, which are held rigidly in position, leaving a cavity under each pipe or other embedded item. This cavity might at first be filled with water; later with nothing but air. Thus there would be a path for radiation leakage under each embedded item. It is thus necessary to consider bleeding in designing the mix, and to hold this property to the minimum.

The ASTM tentative standard test (C232)—Bleeding of Concrete—involves using half a cubic foot of concrete to measure this property alone. With only the minimum of material available, a modified test was made up, based on the ASTM standard, but using a cylinder molded for the strength test. For measuring bleeding, therefore, the 6" x 12" steel mold for one compression cylinder from each mix was left a half-inch low. Every 15 minutes after casting, the mold was tipped for two minutes, and the bleeding water that collected at one side was transferred to a graduate. Total bleeding water was then prorated into the total water originally in the concrete in the cylinder, and expressed as a percentage for comparison among mixes.

Some direct comparisons can be made from Table III. Mixes 1 and 2 show the effect of adding a densifier; bleeding was increased 50%. Mixes 11 and 12 show the effect of entraining air; although less water was used when air was entrained, bleeding was increased 70%. Mixes 8 and 9 show the effect of simply using less water; bleeding was reduced 45%. Mixes 12 and 13 show the effect of adding -100 fines; even though an extra gallon of water was used, bleeding was reduced by 42%. Mixes 14 and 15 show that when a finely divided diatomaceous earth was substituted for an equal volume of cement; even though an additional 2 gallons of water was used in a one-yard batch, bleeding was cut by 21%. The conclusions are that increasing the fluidity of the paste increases bleeding, and increasing the fines in the mix decreases bleeding.

Strength

Strength was determined by compression tests of 6" x 12" cylinders cured at 70° F and 100% humidity until tested, in accordance with ASTM C39—Compressive Strength of Molded Concrete Cylinders. Fig. 6 shows the relationship between strength and the water-cement ratio by weight for all the cylinders tested, for ages 7, 28 and 90 days. It appears that a water-cement ratio of 0.71 will give 3000 psi concrete at 28 days. The expected rate of strength gain for three water-cement ratios yielding 2000, 3000, and 4000 psi at 28 days is shown on Fig. 7.

In performing one compression test, loading was continued until the cylinder was completely shattered. This cylinder broke at about 3400 psi, and in

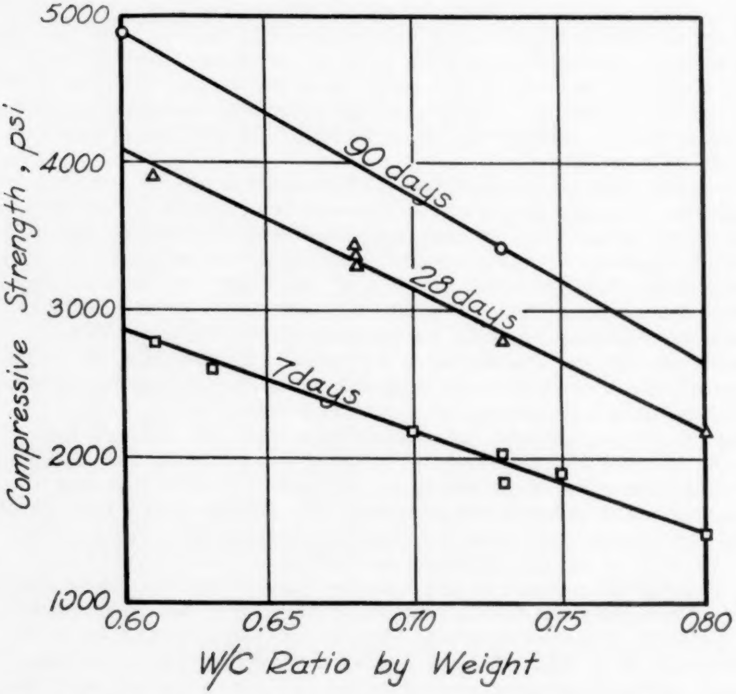


Fig. 6 - Effect of Water-Cement Ratio on Strength

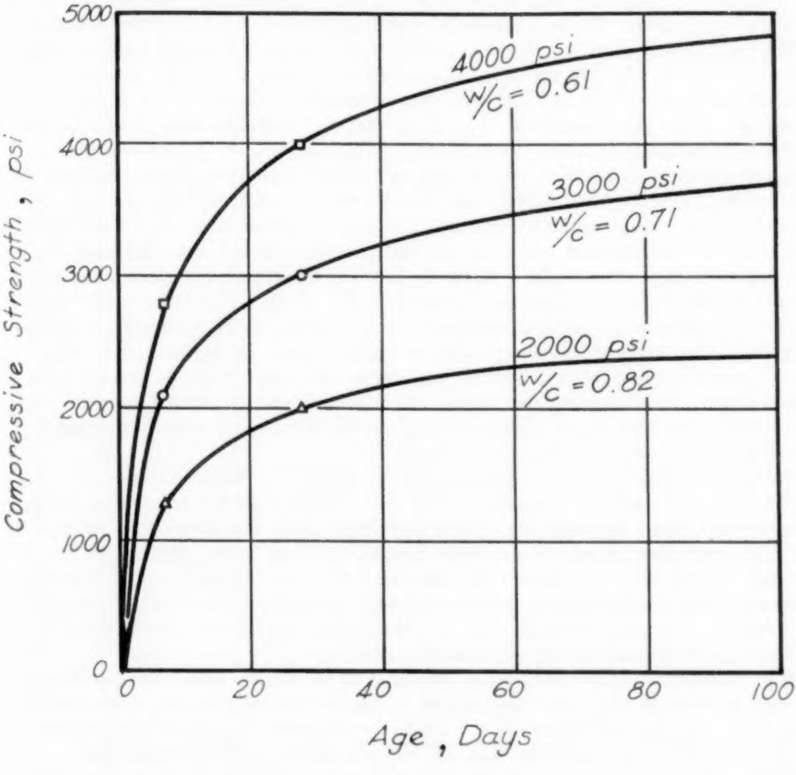


Fig. 7 - Rate of Strength Gain

only three or four places did the break run through an aggregate particle, indicating that the magnetite has strength somewhat greater than that of the hardened cement paste.

Modulus of Elasticity and Poisson's Ratio

Modulus of elasticity and Poisson's ratio was determined on two series of 6" x 12" cylinders. Three cylinders were tested at a time, and the results averaged. Tests were made for the 40% sand and the 45% sand series, using Mixes 9 and 15. As yet there is no standard ASTM test for the elastic properties of concrete, but the procedures used for these tests are representative of the best practice. For each test, the cylinder was preloaded to half the ultimate load, then the load backed off to 1500 lb. Load was then increased steadily to failure, at the rate of 10,000 lb per min, with longitudinal and lateral deformations being read every 2500 lb until well beyond the elastic range. Longitudinal deformations were read on an 8-inch compressometer with a multiplying factor of 2.0 which averaged the deformations on two opposite elements of the cylinder. Lateral deformations were read from a dilatometer with a multiplying factor of 2.27, which indicated the increase in the middiameter of the cylinder as it was loaded. Both instruments were fitted with dial indicators which read to 0.0001-inch. Stresses and strains were plotted against each other for all these tests, and slope of the straight line portion of the longitudinal strain curve was taken as the modulus of elasticity. For this same portion of the curve, the lateral strain divided by the longitudinal strain is Poisson's ratio.

Results of both series are assembled on Table IV. Both modulus of elasticity and Poisson's ratio are about 50% higher than for concretes made with normal stone aggregates. From previous tests, it has been noted that the corresponding constants for light-weight concretes are lower than for normal concretes. This seems to indicate a distinct variation of the elastic constants with the density of the aggregates; the more dense the aggregate, the greater it resists deformation, and the higher will be the modulus of elasticity and Poisson's ratio of the concrete.

Fig. 8 indicates the variation of modulus of elasticity with time for three strengths of magnetite concrete, based on experimental results. For design purposes, it is recommended that a modulus of elasticity of 4.5 million psi be used for 3000 psi concrete, and 6.0 million psi be used for 4000 psi concrete. For all concretes in the 220-240 pcf density range, regardless of strength, it is recommended that the Poisson's ratio for design purposes be 0.25.

Shrinkage and Coefficient of Thermal Expansion

Specimens for the determination of drying shrinkage and coefficient of thermal expansion were 4" x 4" x 40" bars, fitted with brass contact points at the ends. For each of Mixes 9 and 15, six such bars were cast, and cured for 28 days at 70° F in the fog room. At the end of this time, three bars of each mix were measured, weighed, and removed to storage at 70° F and 50% humidity to determine drying shrinkage. The remaining three bars of each mix were dried in air one day, and then coated with two coats of neoprene primer, and three coats of neoprene rubber. Following this, they were given

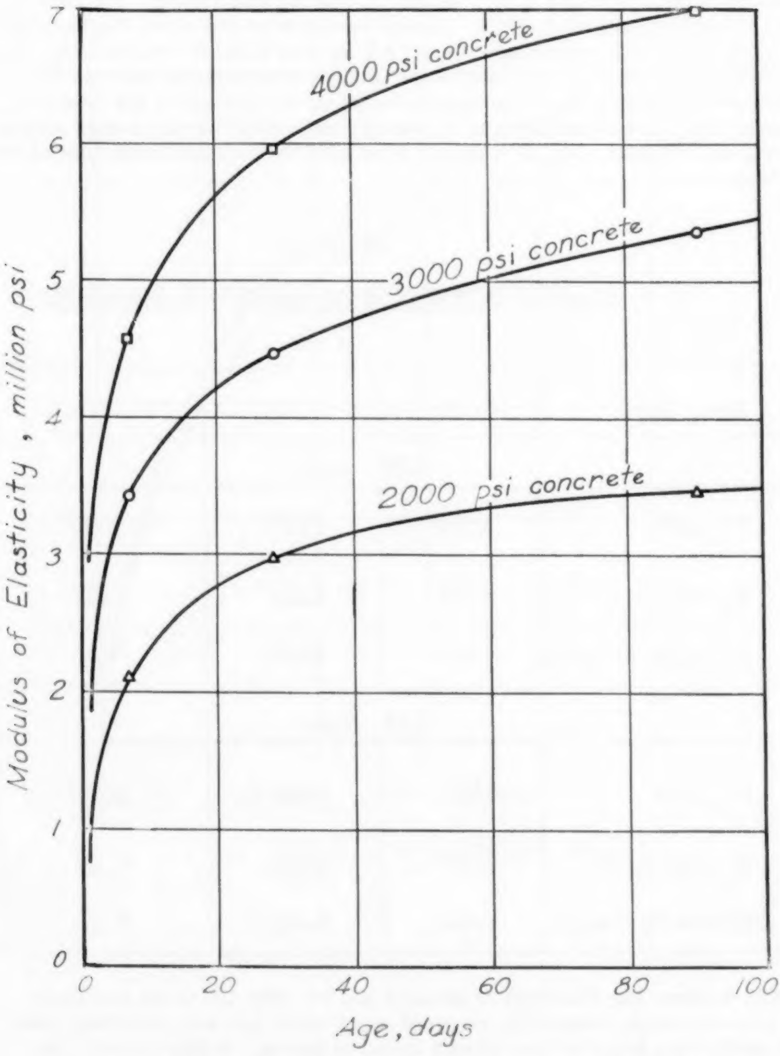


Fig. 8 - Modulus of Elasticity

two complete storage cycles from 100° F to 40° F, with lengths being measured after two days of storage to equalize temperatures. From these temperature cycles, it was determined that the coefficient of thermal expansion of Mix 9 was 4.8×10^{-6} in/in/° F, and for Mix 15 was 5.1×10^{-6} in/in/° F for the range 40 to 70° F. Higher values were measured for the range from 70 to 100° F, being respectively, 5.7 for Mix 9 and 6.1 for Mix 15. This increase in coefficients has been noted in other concretes(14) and is considered to be a function of the moisture content and the age of the concrete. Considering all factors involved, it was recommended that for design purposes a mean value of 5.0×10^{-6} in/in/° F be used for thermal coefficient of expansion.

TABLE IV

ELASTIC PROPERTIES OF MAGNETITE CONCRETE

Age, Days	7	28	90
40% Sand			
f'_c, psi	1820	2790	3430
E_c, psi x 10^{-6}	3.12	4.49	5.37
Poisson's ratio	----	0.26	0.27
45% Sand			
f'_c, psi	1850	3060	4010
E_c, psi x 10^{-6}	2.89	4.30	5.49
Poisson's ratio	----	0.24	0.26

Fig. 9 shows the shrinkage of Mixes 9 and 15. Mix 15, which contained the fine pozzolanic admixture, required more water and was shrinking about 10% faster than Mix 9 at the end of a month of drying. In this respect, the mixes are acting normally, as shrinkage increases with the amount of water used in the mix. The shrinkage of about 0.06% at the end of a year is slightly lower than average. This must be due in large part to the influence of the magnetite itself, since aggregates having a high modulus of elasticity and rough surfaces offer greater restraint to shrinkage than aggregates having low modulus of elasticity and smooth surfaces.

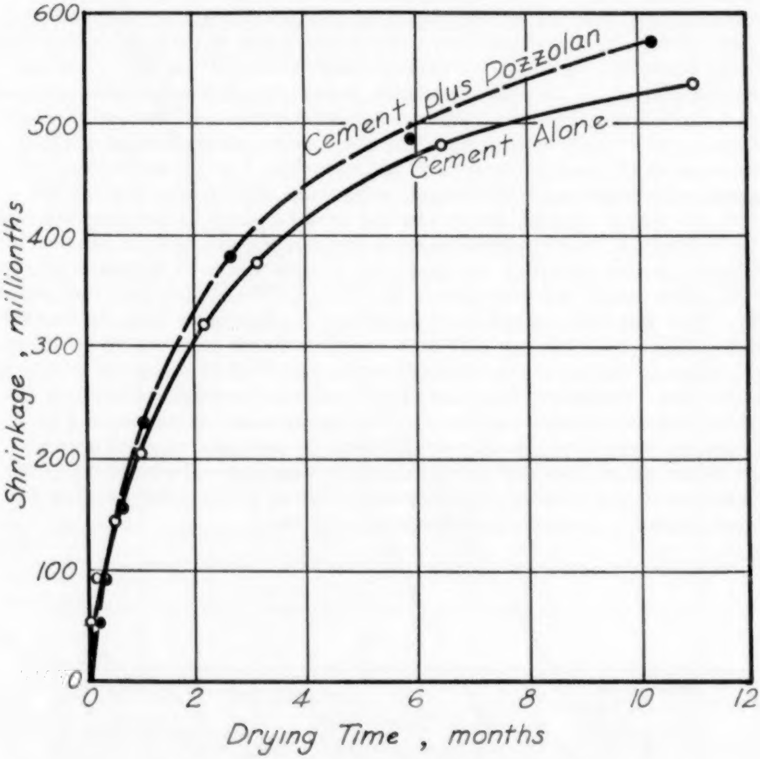


Fig. 9 - Shrinkage

Radiation Shielding

While it is possible to compute a theoretical value for the absorption of gamma radiation by magnetite concrete, as for any material, if its exact composition is known, it was deemed desirable to make a direct determination of the mass absorption coefficient and the build-up factor, using concrete specimens fabricated during the course of this investigation. Accordingly, 3" x 6" and 6" x 12" cylinders were sawed into slabs of varying thickness, so that they could be combined into various lengths from 2" to 12". Two test setups were used, one of which is shown in Fig. 10. In a narrow beam experiment, a Co^{60} source produced gamma radiation which was beamed through a 1/2" diameter 4" long collimator. The beam then passed through varying thicknesses of 3" diameter concrete, and thence to a scintillation counter. The relative attenuation of the radiation by the concrete over the normal count in air was computed, divided by the actual density of the concrete, and plotted in Fig. 11. A slightly different arrangement was used to produce a wide beam, as shown in Fig. 10. Here the source was a 4" diameter piece of blotting paper which had been saturated with a Co^{60} solution and then allowed to dry. This was then placed in the bottom of a glass petri dish, so that the radiation passed out through a 3" diameter hole in the lead bricks as shown. With a beam of this strength, thicknesses up to 12" of 6" diameter cylinders could be used. Results of this part of the test were computed similarly to the above and also plotted on Fig. 11. The broad beam as well as the narrow beam attenuation curve plot as straight lines on this semi-logarithmic plot within the values of absorber thicknesses investigated. The build-up factor B is also linear but it is believed that the build-up factor will approach a constant value for greater thicknesses of concrete.

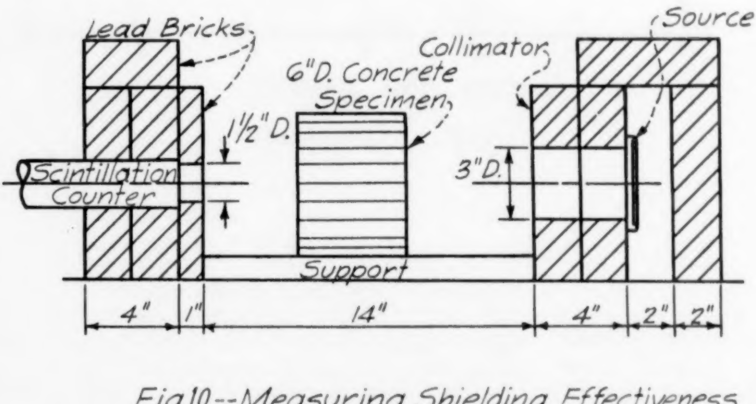


Fig.10--Measuring Shielding Effectiveness

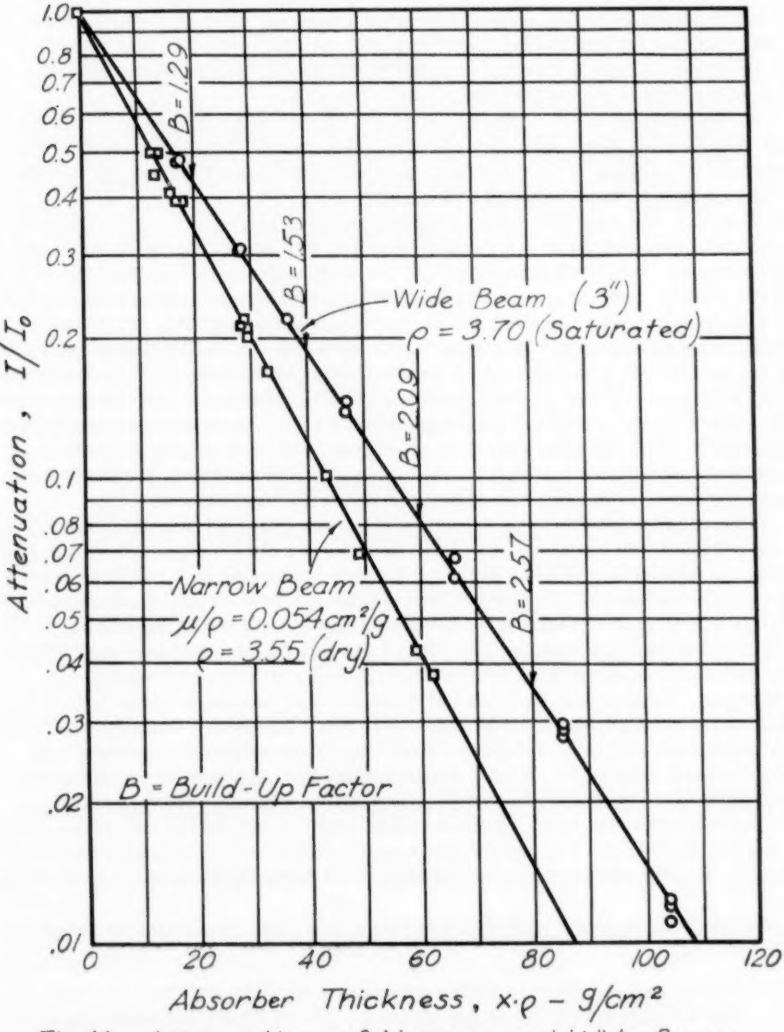


Fig. 11 - Attenuation of Narrow and Wide Beam Co^{60} Gamma Radiation by Magnetite Concrete

Field Experience with Heavy Concrete Mixes

Very close control of all stages of production were necessary in order to produce uniform high quality concrete. The magnetite was mined in Nevada, approximately 28 miles from the nearest railroad. The ore was blasted, loaded on trucks and hauled to a 36" x 40" crusher where it was reduced to 4" maximum size. It was then hauled in 12 cu. yd. trucks to the railroad siding where it was recrushed and screened to size.

At the start of operations, some difficulty was encountered in securing the required apparent specific gravity of 4.4, due to the presence of abnormal amounts of gangue. This particularly affected the sand fraction, as the gangue broke up more readily than the magnetite.

The magnetite as laid down in nature reduced in richness and consequently in density either side of the seam in which it had been originally deposited. Preliminary control of density of the ore was made by controlling the location where the ore was mined. A more uniform product was secured by the use of magnetic rolls to separate the gangue from the magnetite. Shipment of the sized magnetite across two states was done in 50-ton bottom dump cars. A degradation of less than 1% occurred in transit, evidenced by the increase in the fine sizes. However, the additional fines tended to accumulate in pockets and caused some trouble in maintaining a uniform mix. Stock piles of the aggregate at the job site were placed on concrete pads to insure their being kept clean and to reduce waste. Two days prior to batching, the stock piles were wet down in order to stabilize the water absorption.

Considerable trouble was experienced in the concrete mixing plant due chiefly to the increased weight of the materials used. The plant, which was designed to handle material weighing 110 pcf loose was now handling magnetite which weighed 170 to 180 pcf loose. All material handling equipment required adjustment to handle this heavier material. The non-operating time due to plant failures amounted to 40% of the total time. Excessive vibration was noted and attributed to both the weight and surface roughness of the magnetite. This vibration loosened bolts, caused bearings to run hot, and mountings and equipment drive-belts to break. Excessive wear was noted on the shaker screens and standard sieves due to the angular roughness and weight of the magnetite. Power supply which was adequate for ordinary concretes was inadequate to run the plant at full capacity for magnetite.

The concrete mix as designed vibrated readily and moved out to completely fill the forms. In wall forms, successful pours without segregation were made both using trunks or dropping freely. No excessive water gain was noted.

The field experience with this magnetite concrete may be summarized as follows:

1. Water-cement ratio is very critical and must be watched for each batch if the aggregate is not uniform. Close liason between inspectors at the pour and those at the plant is necessary.
2. Water for maximum density is critical. Therefore, special attention must be given to mixers, transit trucks, and hoppers to insure they be not overloaded, and vibration should be provided to unload hoppers.
3. If the concrete has relatively slow setting time, allowance should be made for this and for the extra weight in designing form work.
4. With harsh and variable magnetite aggregate, 5.5 scy was used to provide the fat mortar which supported the aggregate without segregation

and at the same time gave the specified weight of 220 pcf. In the field 5.0 scy resulted in segregation and loss of weight upon vibration.

5. Admixtures intended to increase workability actually resulted in much segregation and loss of weight.

CONCLUSIONS

Magnetite concrete for reactor shielding can be successfully mixed and placed by conventional methods if careful attention is given to the design of the concrete mix. The heavy aggregate particles must be floated in a dense, tough cement paste if undue bleeding and segregation are to be avoided. ASTM standard tests can be used as a guide or modified as necessary in order to secure the structural properties of high-density concrete.

ACKNOWLEDGMENT

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DESIGN OF MASONRY WALLS FOR BLAST LOADING

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(Proc. Paper 1512)

ABSTRACT

The arching action theory of masonry walls presented in a previous paper is applied to the problem of blast resistant design. Based on this theory, an equation of motion is developed for a masonry beam of solid cross-section restrained by essentially rigid supports. This equation is solved for a simplified but realistic form of blast loading. The results obtained permit either the design of a wall for a given loading or the determination of the maximum loading which a given wall can withstand. The theory is extended by an approximate method to wall panels supported on four sides. Comparison is made with test data for walls subjected to full scale atomic and high explosive blasts.

INTRODUCTION

In a previous paper a theory was developed to explain the increased resistance of certain unreinforced masonry wall panels to statically applied lateral loadings. According to this theory, which is applicable to masonry panels constrained between essentially rigid supports, the resistance of the panel is due to forces developed in the plane of the panel as the masonry material tends to be crushed at midspan and the end supports. The present paper applies this theory to the problem of the response of a masonry[3] wall

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3. The term masonry is used to include all unreinforced walls constructed of brick, concrete, stone, cinder block, rubble, etc.

to dynamic forces, such as those which arise from an atomic blast. The results obtained are presented in a form suitable for the design of masonry walls for given blast loads, and comparison is made with available experimental data.

Mode of Response According to the Arching Theory[4]

Consider a masonry beam constrained between non-yielding supports at its lateral edges. Several assumptions are made: (1) the beam has a uniform, solid rectangular cross-section, with span L, depth d, and unit width (see Fig. 1) (2) the stress-strain curve of the material in compression is elastic up to a limiting (crushing) stress s_c (and corresponding strain, e_c), (3) for compressive strains greater than e_c, the stress remains constant at s_c, or what is equivalent, the material crushes at a constant state of stress, (4) the material possesses zero tensile stress, and (5) the material exhibits no strength recovery beyond the elastic range. A complete load-deflection cycle for masonry according to the above assumptions of stress-strain behavior is shown in Fig. 2.

The assumed mode of response of the beam may be described as follows. Immediately upon loading, cracks develop on the tension sides at the ends and center of the span. Initially, these cracks extend to the center-line of the beam. During subsequent motion it is assumed that each half of the beam remains rigid and rotates about its end support and the center. The resistance to this rotation is developed by a force couple acting at the ends and center as a result of the crushing of the masonry at these locations, see Fig. 1. The reference to "arching action" results from the resemblance of this configuration to that of a three-hinged arch.

Equation of Motion

Consider a uniformly distributed, time-dependent pressure, p(t), acting on the beam as shown in Fig. 1. According to the assumed mode of response, each half-span rotates as a rigid body about the point of contact at the support. Utilizing the D'Alembert approach, the equilibrium condition for the half-span (of unit width) can be written as

$$\frac{L^3 \gamma}{24g} \ddot{\theta} + M(\theta) - \frac{L^2}{8} p(t) = 0 \qquad (1)$$

where

θ = angular displacement of the beam, see Fig. 1

L = span of beam

γ = weight of masonry beam per unit length per unit width

g = gravitational constant

$M(\theta)$ = rotation-dependent moment resistance due to arching action

4. This section contains a brief summary of the arching theory presented in reference 1. However, the present nomenclature has been modified somewhat.

p(t) = applied time dependent force per unit length on a unit width of beam

t = time

$\ddot{\theta}$ = $\frac{d^2\theta}{dt^2}$ = angular acceleration

Denote the center deflection by w. Then, for small displacements,

$$\theta = \frac{w}{L/2} \tag{2}$$

It is convenient to introduce a non-dimensional center deflection, u, defined as

$$u = \frac{w}{d} \tag{3}$$

where d is the depth of the beam. Using Eqs. (2) and (3), Eq. (1) may be re-written in the following non-dimensional form:

$$m\ \ddot{u} + k\ (u) = f\ (t) \tag{4}$$

where

$$m = \frac{4}{3g}\frac{L^2}{s_c}\frac{\gamma}{d}$$

$$k(u) = \frac{16\ M\ (u)}{s_c d^2} \tag{5}$$

$$f(t) = \frac{2L^2}{s_c d^2}\ p\ (t)$$

The analytical form for the moment resistance due to arching, M(u), was developed in reference 1. The non-dimensional resistance, k(u), is shown in Fig. 3 as a function of the family parameter, R, where

$$R = \frac{e_c L^2}{4d^2} \tag{6}$$

Blast Loading

Complex patterns of shock waves are generated when atomic weapons are detonated. While it is not the purpose of this paper to consider in detail the prediction of dynamic forces imposed on structures by blasts, specific load forms are required. The forms described below are based on published data.

It is assumed that the structure is windowless, unshielded by other structures, and loaded head-on by a single, essentially plane shock. For structures of usual size located at sufficient distances from the center of the explosion, consideration of a single, plane, shock is valid. Also, the maximum load on the front wall of a structure is developed when the wall is unshielded and oriented parallel to the shock front (head-on loading). In the general (or random) case, the wall would be oriented obliquely to the shock so that its load would be less than the maximum. The design procedure outlined later for this maximum load will therefore be conservative.

A general description of shock phenomenology associated with nuclear detonations is given in References 2 through 5. It may be helpful though to

review a few basic notions. The strength, or intensity, of the shock is a function of the atmospheric pressure in the undisturbed regions of the shock and the pressure in excess of atmospheric just behind the shock. The latter pressure, which is termed the peak overpressure, is denoted by p_σ. At a given distance from the explosion the overpressure decays with time from its peak value to zero, becomes negative, and then returns to zero at a relatively slow rate. The first period of loading is termed the positive phase and its duration is denoted by t_o; the second period is called the negative phase. Negative phase loads are generally ignored in structural response analyses because the forces are appreciably less than those developed during the positive phase.

The variation in blast pressure on a front wall under the above conditions may be represented by Fig. 4. With reference to Fig. 4, the pressure is reflected to the value p_r at zero time, i.e., the time at which the shock first strikes the wall. The pressure p_r depends on the peak overpressure p_σ and on atmospheric conditions; the value given in Fig. 4 is for an atmospheric pressure of 14.7 psi. The reflected pressures decrease at a rate which is generally assumed to be constant up to the so-called clearing time $\frac{3h}{U}$. The dimension h, called the clearing distance, depends on the geometry of the structure.[5] It may be taken as either the height or the half-width of the structure containing the wall, whichever is the smaller. The quantity U is the average speed of the shock front and, based on one-dimensional shock wave theory, can be given in terms of overpressure as:

$$U = 425\sqrt{7 + \frac{6\,p_\sigma}{14.7}} .$$

The loading period up to the clearing time is often referred to as the diffraction phase. The subsequent loading period up to the positive phase duration, t_o, is sometimes known as the pseudo-steady-state or drag phase, and takes its name from the analytical form normally associated with steady-state drag loadings. Generally accepted expressions(3) for the overpressure time variation, $p_\sigma(t)$, the drag or dynamic pressure-time variation, $p_d(t)$, and the front wall drag coefficient, C_{df}, are the following:

$$p_\sigma(t) = p_\sigma e^{-t/t_o}(1 - t/t_o)$$

$$p_d(t) = p_d e^{-2t/t_o}(1 - t/t_o)^2$$

$$p_d = \frac{2.5\,p_\sigma^2}{103 + p_\sigma} \quad \text{(for 14.7 psi atmospheric pressure)} \qquad (7)$$

$$C_{df} = 1.0$$

5. It is assumed here that at least one dimension of the wall panel under consideration is essentially the same as the structure, as would be the case for many single-story buildings.

Simplified Front Wall Loading

The loading shown in Fig. 4 is useful in the sense that it provides a good measure of the true value of the blast load. However, the analytical form in which this loading is expressed can not be incorporated readily into the equation of motion, Eq. (4). To facilitate a solution of Eq. (4), the loading is simplified further.

To justify the following simplification, it will be helpful to consider the time scale indicated in Fig. 4. For a representative panel with clearing distance h = 12 ft and shock speed U = 1350 ft per sec (corresponding to a shock of approximate 8 psi overpressure), the clearing time is $\frac{3h}{U} = 0.027$ seconds. For the so-called nominal bomb, i.e., 20 kilotons equivalent of TNT, such as that used in Japan, the positive phase duration, $t_o = 0.6$ sec. Now, experience indicates that if a masonry panel were to fail structurally under this loading, it would do so in about 0.05 seconds or less. From these values it can be concluded that the loading on the wall remains essentially constant between the clearing time and the failure or break time, and that at this latter time $p_\sigma(t)$ and $p_d(t)$ do not differ much from their initial values p_σ and p_d. Thus, the load form shown in Fig. 5a may be assumed to apply in those cases where the loading is associated with failure of the wall panels. A further simplification may be made by replacing the load form in Fig. 5a by the form in Fig. 5b.[6] Part of the diffraction impulse (the shaded area in Fig. 5a) is considered to be applied to the wall as an initial impulse, i_o. It may be noted that from the point of view of design, this assumption is conservative. The subsequent loading then has the constant value, $p_o = p_\sigma + p_d$.

The effect of the initial impulse, i_o, is to impart an initial velocity, $\dot{u}_o$, to the wall which is proportional to i_o. The constant pressure p_o and the impulse i_o are plotted against overpressure in Fig. 6. As will be shown in the following section, these load simplifications lead to a particularly convenient solution of the equation of motion.

Solution of the Equation of Motion

Based on the simplified load form of Fig. 5b, Eq. (4) may be written

$$m\ddot{u} + k(u) = f \tag{8}$$

subject to the following modified initial conditions,

$$\text{at} \quad t = 0,\ u = 0,\ \dot{u} = \dot{u}_o = \frac{I}{m} \tag{9}$$

where

$$f = \frac{2L^2}{s_c d^2} p_o$$

$$I = \frac{2L^2}{s_c d^2} i_o \tag{10}$$

6. Equation (4) was solved numerically for the load form shown in Fig. 5a and the results obtained in representative cases differed from solutions based on the simplified load form of Fig. 5b by less than 10 percent.

and m is defined in Equation (5). Using the identity

$$\ddot{u} = \dot{u}\frac{d\dot{u}}{du} = \frac{d}{du}\left(\frac{1}{2}\dot{u}^2\right),$$

Eq. (8) may be integrated to yield

$$\frac{1}{2} m \left[\dot{u}(t)\right]^2 - \frac{1}{2} m\dot{u}_o^2 = \int_o^{u(t)} \left[f - k(u)\right] du \tag{11}$$

Let the maximum displacement u_c occur at time t_c, i.e., $u(t_c) = u_c$ and $\dot{u}(t_c) = 0$. Then Eq. (11) yields

$$\frac{1}{2} m \dot{u}_o^2 = \int_o^{u_c} k(u)\, du - fu_c \tag{12}$$

The displacement u_c has so far been interpreted simply as a maximum displacement. It will now be shown that Eq. (12) implies that for any given value of f, there exists a $\dot{u}_o$ which determines a critical maximum displacement; conversely, for any $\dot{u}_o$, there exists a value of f which determines this displacement. The displacement is termed "critical" because an infinitesimal increase in f or $\dot{u}_o$ produces a displacement that increases without bound. Thus one can determine related maximum values of f and $\dot{u}_o$ which a given wall can safely withstand.

The above statements are conveniently established with reference to Fig. 7 in which the individual terms of Eq. (12), having units of work or energy, are plotted against the displacement u. Note that the initial kinetic energy, $\frac{1}{2} m \dot{u}_o^2$, is represented by the ordinate OA, that the straight line segment AB has slope equal to f, and that the ordinate BC of the curve, S, is equal to the integral term of Eq. (12). It is seen that for a given value of $\frac{1}{2} m \dot{u}_o^2$, the maximum value of f yielding a real value of u_c is that for which the line segment AB is tangent to the curve S at point B. Since B is a point of tangency, it is concluded that at the displacement u_c, $f = k(u_c)$; this might have been inferred at the outset from static considerations alone.

The results of the computations indicated above are shown in Fig. 8 in which related maximum permissible values of p_o and i_o (which define f and $\dot{u}_o$ through Eqs. 9 and 10) are plotted against the parameter R, see Eq. (6); values of the critical displacement u_c are also shown in the figure. These results were obtained for each value of R by first specifying a value of f, finding u_c through the relation $f = k(u_c)$, and then computing $\dot{u}_o$ by means of Eq. (12).

Two-Way Panels

Previously only the response of a beam or one-way panel, i.e., a panel supported on two opposite edges, has been considered. In many cases, of course, one must consider panels supported on four sides. The extension of the arching action theory of masonry wall behavior to two-way panels appears to be rather involved and to date it has not been accomplished. Therefore an approximate solution of this problem in terms of a so-called equivalent length of beam is proposed, to serve as an interim guide for designers.

The concept of an equivalent beam is commonly employed in the design of two-way reinforced concrete slabs.[6] The equivalent beam in this case has the same length as one side of the panel, but its loading is modified to compensate for "two-way behavior". In masonry wall design it is more convenient to retain the loading and determine an equivalent beam length. This equivalent length is determined so that a given uniform, static, load will produce the same center (maximum) deflection in the beam as in the panel.

Equivalent beam lengths were computed on the basis of two separate assumptions for the action of the two-way panel. The results are shown in Fig. 9 where the equivalent beam length, L, is plotted against the ratio of the actual panel dimensions, L_1/L_2 ($L_1 \leqslant L_2$). The curve labeled "elastic" theory is based on the assumption that the panel deflects like a simply supported homogeneous elastic plate;[7] the curve labeled "yield-line" theory is based on the assumption that the panel deflects according to the yield-line failure theory.[8] Since these curves agree fairly well (which is rather interesting) it seems plausible to use an average of the curves for determining equivalent beam lengths. The presently recommended values, accordingly, are given by the curve labeled "average" in Fig. 9.

Comparison with Tests

Atomic Test Data

Much of the data derived from full scale testing of masonry panels subjected to atomic blasts under controlled conditions is regarded as classified information. However, the results of two such tests can be indicated here.

Under the sponsorship of the U. S. Air Force, the Armour Research Foundation designed a wall panel test as part of a weapons effects program several years ago. In this test three identical unreinforced brick masonry walls, supported on four sides, were located at different distances from the center of the explosion. The farthest panel was essentially undamaged, whereas the closest panel was blown out. The intermediate panel remained intact but showed deep cracks and appeared to be on the verge of complete failure.

These results were in excellent agreement with the present theory. In the case of the farthest panel, the load in terms of i_o and p_o (as computed from the measured overpressures) was considerably less than the theoretical critical load. Values of i_o and p_o for the intermediate panel were slightly less than the critical values of i_o and p_o, whereas for the closest panel, the critical values were exceeded by far.

These predictions were based on average material properties and an equivalent beam length. The panels were extensively instrumented to provide data on the forces transmitted to the supporting structure both in the plane of, and normal to, the panel. These data again agreed reasonably well with the arching theory.

Another extensive wall panel test was sponsored by the Federal Civil Defense Administration (FCDA) in this same program. Data which would describe the test structures completely was not available to authors. Nevertheless, an attempt was made to interpret the test results; reasonable guesses were made where exact information was lacking.

Actual values of i_o and p_o (as computed from measured overpressures) were compared with the critical values of i_o and p_o predicted by the arching theory. The actual values were less than the theoretical values in every

case where a wall remained intact, and greater where the walls failed. This, of course, is what one would expect if the theory were correct in all respects.

High Explosive Test Data

A group of masonry wall panels both reinforced and unreinforced, was exposed to high explosive detonations in an experimental program designed and conducted by the Armour Research Foundation for the Structural Clay Products Research Foundation of Geneva, Illinois. The objective of this research program was to investigate the potentialities of structural clay masonry walls for blast resistant design.

One phase of this test program employed a rigid octagonal fixture which held eight wall panels of various types. A high explosive charge was detonated in the center of the fixture, thereby loading each panel in identical fashion. Details of this test are contained in references 9 and 10. The behavior of one panel is pertinent to this discussion.

This panel, which was built of unreinforced eight-inch brick masonry, measured 10 feet horizontally by 8 feet vertically, and was designed to arch in the horizontal direction with essentially negligible restraints in the vertical direction. No deformations were observed after the wall received a measured impulse per unit area of 0.19 psi-sec from a 45 lb charge of explosive.

Characteristically, the blast loading on a wall panel due to a high explosive detonation is of extremely short duration, say, of the order of three milliseconds. It is therefore reasonable to suppose that the effect of this loading is to impart only an initial impulse to the wall. Thus the results of the proposed theory as given in Fig. 8 should be applicable, taking p_o as zero.

The physical properties of the wall tested may be described as follows: $e_c = 0.001$, $L = 120$ in, $d = 8$ in, $\tau = 0.59$ psi, $(R = 0.056)$. The critical impulse, i_o, i.e., the impulse required to produce failure of the wall, is tabulated below for several values of the crushing strength, s_c.

Crushing Strength, s_c psi	Critical Impulse, i_o psi
1000	0.20
2000	0.29
3000	0.35

It is seen that, over a reasonable range of wall strength as characterized by s_c, the predicted impulse required to fail the wall exceeds the measured impulse of 0.19 psi-sec. The present theory thus predicted correctly that the wall would withstand the impulse applied.

It is illuminating to compare this approach with results predicated on elastic behavior of the wall. A critical impulse can be computed on the assumptions that only bending stresses are set up in the wall, and that failure occurs when the ultimate tensile stress is exceeded at the center. For a beam with fixed ends, the critical impulse found in this manner is less than 0.01 psi-sec, even if the masonry material is credited with a tensile strength exceeding 200 psi. Since the applied impulse was nearly 20 times this value, it is clear that the panel could not have derived its strength from the mechanism of simple bending.

It should be made clear that the test results reported are not regarded as proof of the arching theory. Nevertheless, these tests do indicate that the effects of varying certain parameters can be predicted fairly well, and on an over-all basis, the theory seems to give rather accurate results. However, no definitive statements can be made without additional testing and, especially, until more is known concerning the physical properties of masonry.

Design Data

From the point of view of blast resistant design, there are two closely related problems to be considered, both of which can be solved using arching theory. One is the "investigation" problem: to determine the maximum overpressure a given wall can withstand; the other is the "design" problem: to select a wall to resist a given overpressure. Both problems can be solved with the results presented so far, in general, only by means of trial and error procedures. A procedure for the investigation problem is described below; the design procedure involves only an additional step.

Investigation Procedure

The necessary information is contained in Figs. 6 and 8. Fig. 6 relates the pertinent load quantities p_o and i_o to p_σ, the peak overpressure of the blast wave; Fig. 8 gives the related maximum values of p_o and i_o which a given wall can withstand. The method of determining the desired maximum or critical overpressure is to assume a value of p_σ', find the corresponding values of p_o and i_o using Fig. 6, and then verify whether or not these values define the proper point on Fig. 8. In general this latter point will not check, indicating that the wrong overpressure has been assumed.

A systematic procedure would be to determine a value of p_o from Fig. 8 which corresponds both to the proper value of R and the value of i_o associated with an assumed overpressure, p_σ''. This value of p_o, in turn, defines an overpressure, p_σ' through Fig. 6. Thus these two values of overpressure can be compared, and the procedure repeated until $p_\sigma' = p_\sigma''$.

As a particular example of this procedure, let it be required to find the maximum overpressure which can be withstood by an 8-inch solid brick wall measuring 16 feet by 8 feet. Assume that the wall acts as a two-way panel and that the following properties are given: $s_c = 1200$ psi, $e_c = 0.001$, $\Upsilon = 0.65$ psi, h (clearing distance) = 8 feet. Here $L_1 = 8$ feet and $L_2 = 16$ feet. Thus, the equivalent beam length is found from Fig. 9 to be $L = 0.91 \times 8 = 7.28$ feet or 87.4 inches. The parameter R is then computed from Eq. (6) to be 0.030. At this point in the computation a value for the desired critical overpressure, p_σ' must be assumed. Choose $p_\sigma' = 4$ psi as a first guess. Using this value, i_o/h is read off from Fig. 6 as 0.005 psi-sec/ft, and since h = 8 feet, i_o becomes 0.03 psi-sec. Referring now to Fig. 8, the abscissa is computed to be 0.0176, and the ordinate corresponding to R = 0.030 is found to be approximately $\frac{2\,L^2 p_o}{s_c d^2} = 2.7$. From this relation, p_o is computed to be 13.6 psi which, from Fig. 6, corresponds to an overpressure p_σ'' of about 10.5 psi. Since this value does not agree with the assumed overpressure of 4 psi, an incorrect value of p_σ' has been selected. If this procedure is repeated for, say, $p_\sigma' = 10$ psi, the computed overpressure, p_σ'' is

found to be about 8 psi. If, in this fashion, a curve is plotted of assumed, p'_{σ}, versus computed, p''_{σ}, values of overpressure, the two are found to be equal at approximately 8.7 psi. This represents the desired maximum overpressure for the panel, as can be verified by repeating the above procedure starting with this value.

An analogous procedure can be followed for the design problem. A series of calculations of this type were performed for an 8-inch brick wall over a reasonable range of equivalent lengths, crushing strength, and crushing strain in order to determine the influence of the material properties on the critical overpressure. These results are presented in Fig. 10. For convenience the clearing distance, h, was taken equal to the equivalent length, L. It is seen that the critical overpressure is relatively insensitive to values of e_c, but is quite dependent on crushing strength. Unless other information is available, it is recommended that values of s_c = 1000 psi and e_c = 0.001 be used.

To illustrate the influence of the thickness, d, on wall strength, a series of calculations were performed for various equivalent beam lengths and thicknesses of d = 4, 8, 9 and 12 inches. All panels were assumed to have s_c = 1000 psi, e_c = .001 and h = 10 ft. The results obtained are shown in Fig. 11. It is seen that wall thickness has an appreciable effect on the maximum overpressure the panel can withstand. For example, increasing the thickness from 8 to 12 inches, increases the maximum overpressure by about 200 per cent over the range shown.

The results shown in Figs. 10 and 11 provide design data for certain classes of walls. Similar curves can be readily prepared for other conditions from the data presented here. Curves of this type provide the basis for a simplified and rapid method of design.

It should be noted that the theory in its present form is restricted to walls of solid cross-section and without door or window openings. The theory, however, can easily be extended to walls of non-solid cross-section provided the cavities are located symmetrically with respect to the center-line of the cross-section. In this case one can subtract from the resistance of the solid wall (at corresponding displacements) the resistance of another solid wall whose thickness is the same as the specified cavity in the original wall. The problem becomes considerably more complicated when the cavities are not symmetrically located.

The results of this paper cannot be applied directly to walls having window and/or door openings. In such cases both the equivalent beam length and the form of the blast loading must be modified. Consideration of this problem is beyond the intended scope of this paper.

SUMMARY

The maximum blast pressures that a masonry wall panel can withstand have been determined on the basis of the arching action theory of wall response. The results presented stem from a simplified form of blast loading characterized by a constant force and an initial impulse, and are applicable to masonry walls of solid cross-section so supported as to develop arching. The walls are further assumed to have no openings and to be struck head-on by the blast. The load simplifications are realistic for design considerations where the intent is to select a wall capable of withstanding a given intensity of blast load.

The results of this theory are shown to compare favorably with the full scale experimental data which can be reported at this time. While this corroboration is encouraging, considerably more experimental checking is required to adequately substantiate the theory. Currently, the greatest uncertainties stem from lack of information concerning the physical properties of masonry materials.

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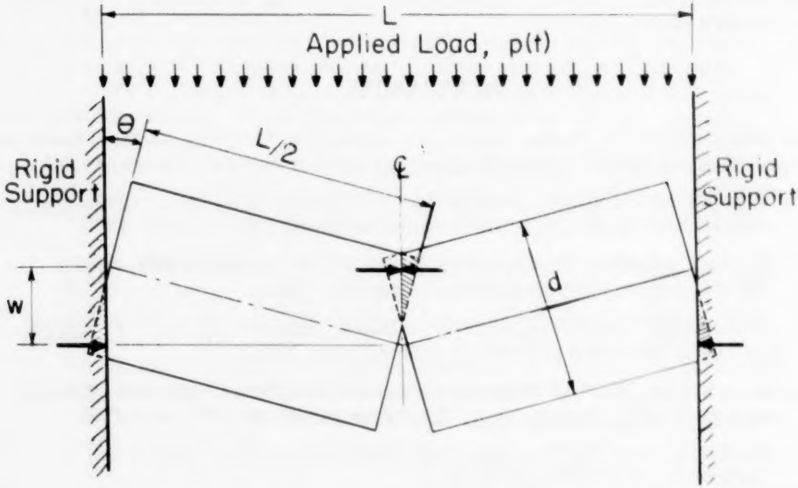

Fig 1 Deflected Position of Idealized Masonry Beam

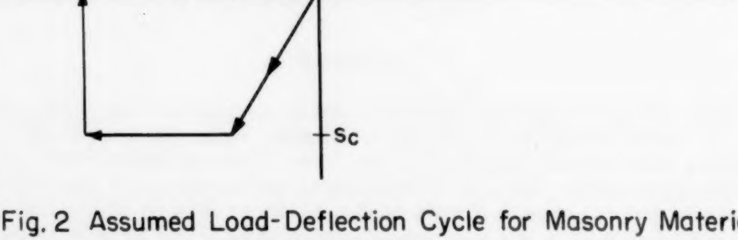

Fig. 2 Assumed Load-Deflection Cycle for Masonry Material

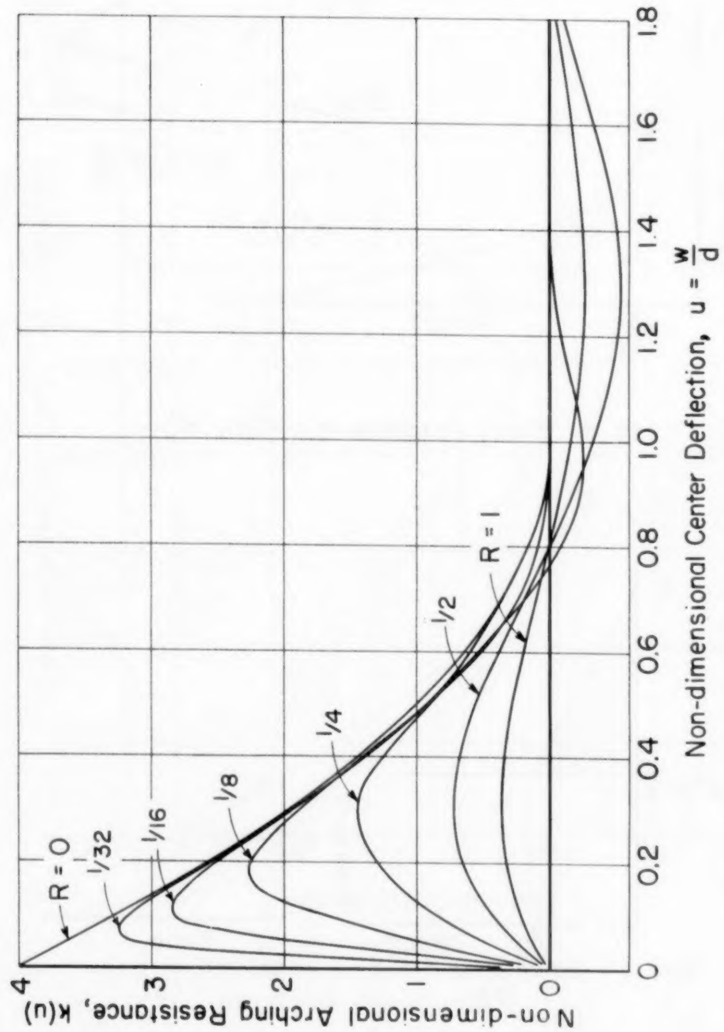

Fig 3 Variation of Arching Resistance with Center Deflection

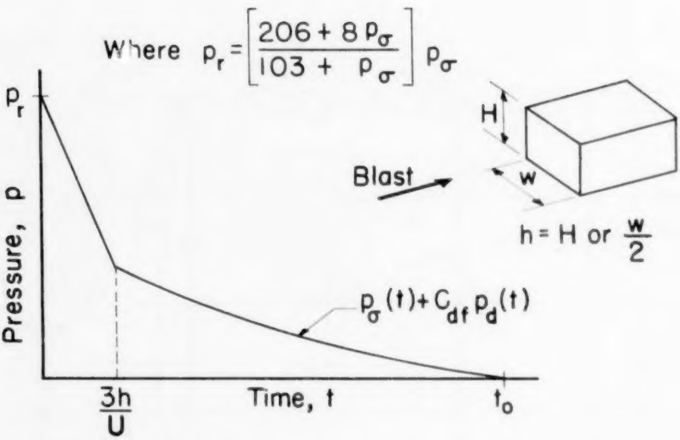

Fig. 4 Blast Pressure on Front Wall

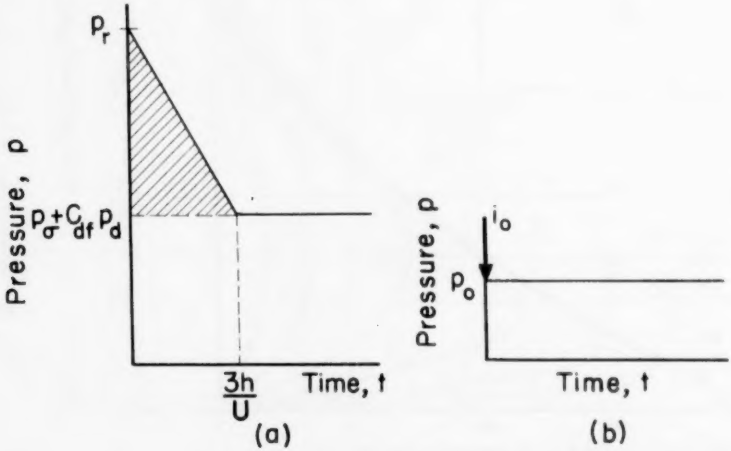

Fig. 5 Simplified Blast Pressure on Front Wall

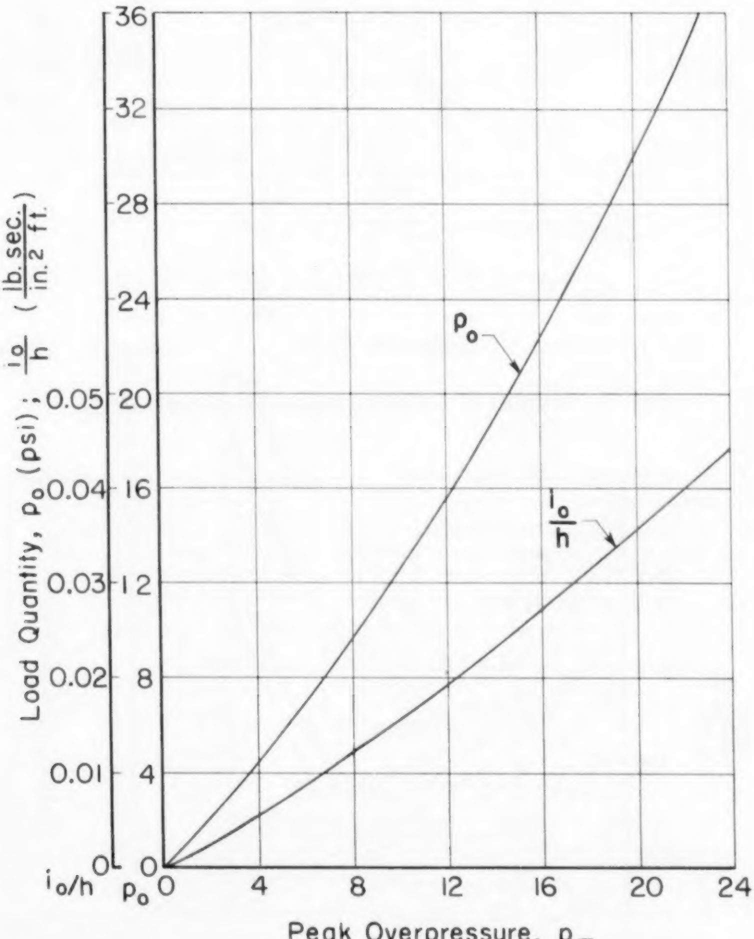

Fig. 6 Variation of Load Quantities, p_o and i_o, with Peak Overpressure

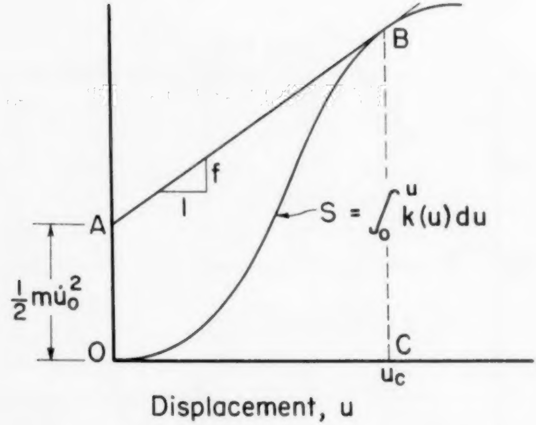

Fig 7 Graphical Interpretation of Equation 12

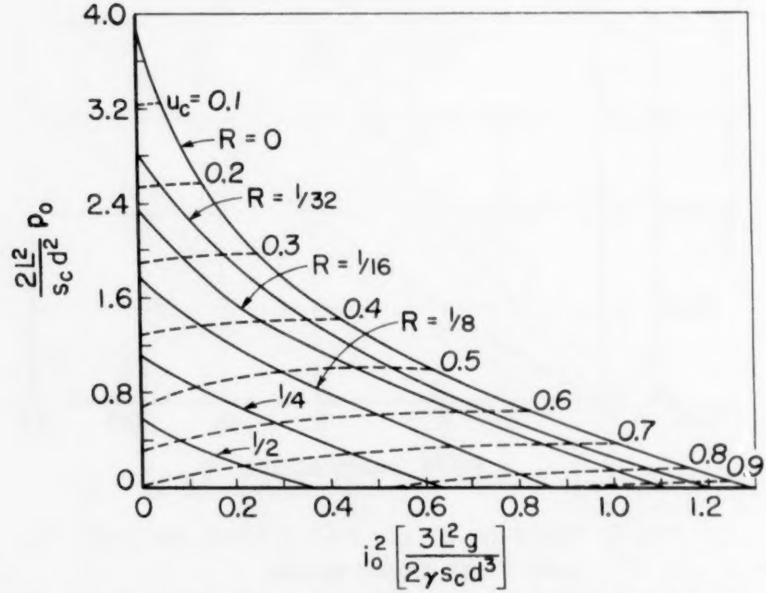

Fig. 8 Maximum Loading (p_o, i_o) which Can Be Sustained by Masonry Panels

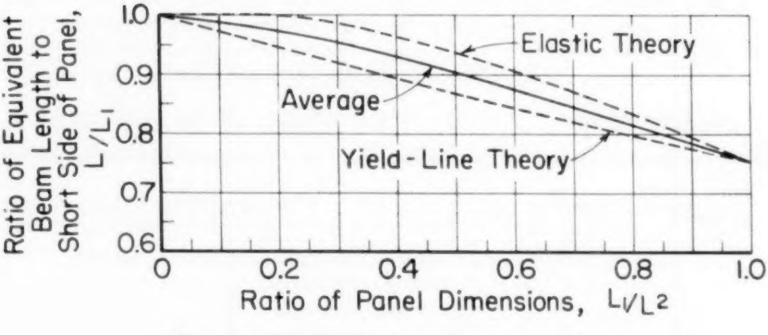

Fig. 9 Equivalent Beam Length

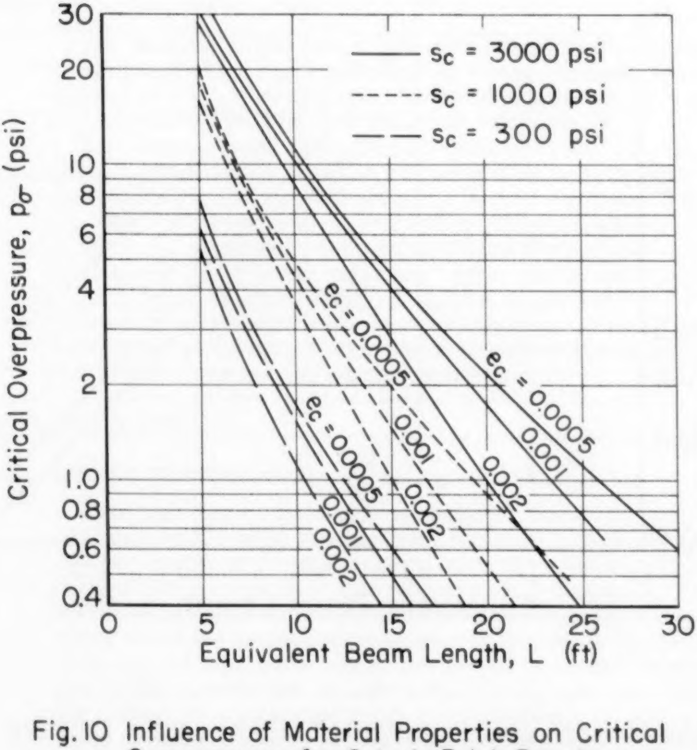

Fig. 10 Influence of Material Properties on Critical Overpressure for 8-Inch Brick Panels

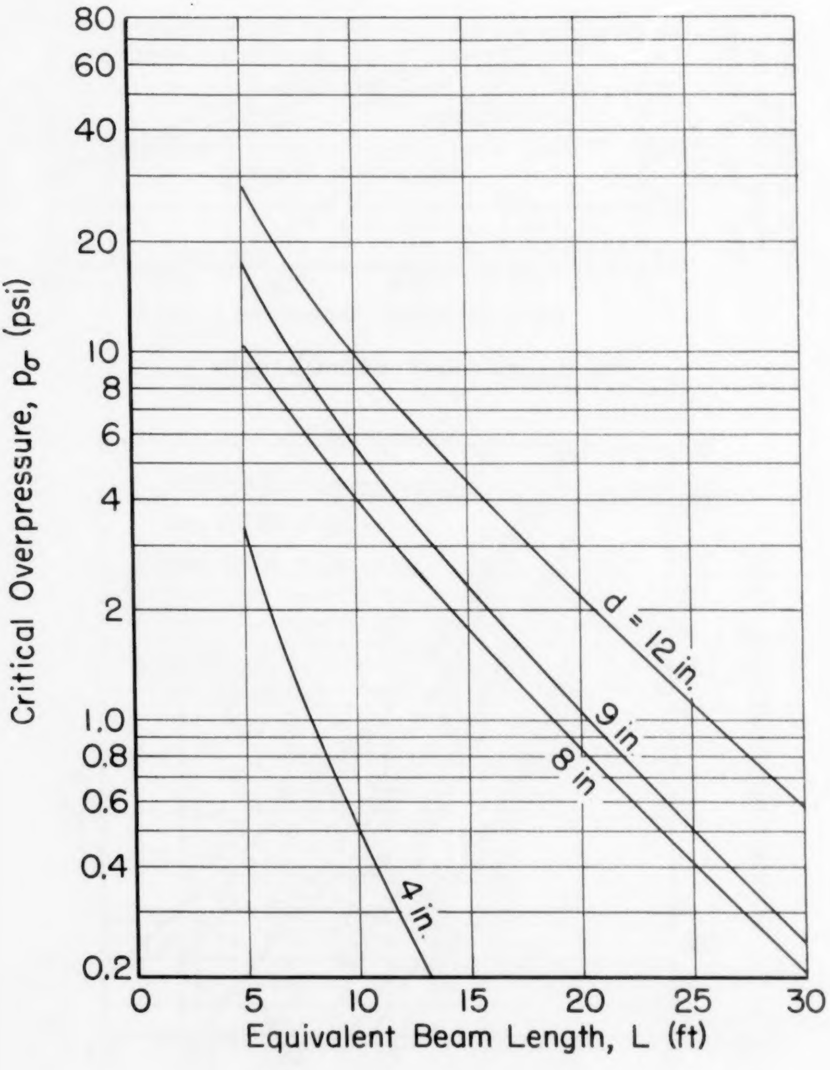

Fig. 11 Influence of Wall Thickness on Critical Overpressure of Brick Panels

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TENTATIVE RECOMMENDATIONS FOR PRESTRESSED CONCRETE

Report of the Joint ACI-ASCE Committee on Prestressed Reinforced Concrete
(Proc. Paper 1519)

CHAPTER 1 - INTRODUCTION

SECTION 101 - OBJECTIVE

The objective of this report is to recommend those practices in design and construction which will result in prestressed concrete structures that are comparable both in safety and in serviceability to constructions in other materials now commonly used.

This report constitutes a Recommended Practice, not a Building Code or Specification. Since it was not written as a Code, its use or interpretation as one will not serve the best interests of either the public or the engineering profession. Recommendations contained in the report are presented solely for the guidance and information of professional engineers. Safety and economy of structures in prestressed concrete will depend as much on the intelligence and integrity of engineers preparing the design and supervising or carrying out the construction as on the degree to which these recommendations are followed.

SECTION 102 - SCOPE

102.1 LINEAR PRESTRESSING

This report is confined in scope to linear structural members involving prestressing with high strength steel; circularly prestressed members such as tanks or pipes are not covered. These types of construction have been excluded for two reasons. They have been designed and constructed in this country for a great number of years and procedures have been developed on the basis of research and experience which have proved successful in practice. Design and construction of tanks and pipes in prestressed concrete are confined to a relatively small group of specialists and are not likely to be attempted by persons outside that group. For these

Note: Discussion open until June 1, 1958. A postponement of this closing date can be obtained by writing to the ASCE Manager of Technical Publications. Paper 1519 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. ST 1, January, 1958.

reasons there seems to be no immediate need for recommendations regarding circularly prestressed structures.

102.2 FLEXURAL MEMBERS

For the most part, recommendations in this report relate to flexural members—beams, girders, and slabs. Other structural forms, such as columns, ties, arches, shells, trusses, pavements, etc., are treated only briefly or not at all. In some of these cases, such as columns or ties, the principles involved in design are essentially simple and no need was felt to include them in this report. In other cases, insufficient information was available either from research or experience to permit recommendations to be made at this time. This lack of information is due in some instances to the complexity of the type of structure involved and in others to the infrequency of its use in this country.

102.3 BUILDINGS AND BRIDGES

These recommendations are intended to apply to both buildings and bridges. The form and nature of this report are such that almost all recommendations made apply without differentiation to both types of structures. Where this is not the case, separate recommendations are made for buildings and bridges.

SECTION 103 - ACCEPTANCE TESTS

It is recognized by the committee that unusual types of construction, design or materials may be used in such a manner that these recommendations are not applicable or may not have been complied with. Such structures may be adequate for the purpose intended. In these cases it is recommended that tests be made to verify design.

SECTION 104 - NOTATION

104.1 GENERAL

Symbols are assembled into sections pertaining to groups of associated terms. The list comprises only the symbols in this report. No attempt is made to present a complete notation for design of prestressed concrete.

104.2 DIMENSIONS AND CROSS SECTIONAL CONSTANTS

A_b - bearing area of anchor plate of posttensioning steel.

A_c - maximum area of the portion of the anchorage surface that is geometrically similar to and concentric with the area of the bearing plate of posttensioning steel.

A_s - area of main prestressing tensile steel.

A_s' - area of conventional tensile steel.

A_{sf} - steel area required to develop the ultimate compressive strength of the overhanging portions of the flange.

A_{sr} - steel area required to develop the ultimate compressive strength of the web of a flanged section.

A_v - area of web reinforcement placed perpendicular to the axis of the member.

b - width of flange of a flanged member or width of a rectangular member.

b' - width of web of a flanged member.

d - distance from extreme compressive fiber to centroid of the prestressing force.

I - moment of inertia about the centroid of the cross-section.

j - ratio of distance between centroid of compression and centroid of tension to the depth d.

p - A_s/bd; ratio of prestressing steel.

p' - ratio of conventional reinforcement.

pf'_s/f'_c - percentage index.

s - longitudinal spacing of web reinforcement.

t - average thickness of the flange of a flanged member.

Q - statical moment of cross-section area, above or below the level being investigated for shear, about the centroid.

104.3 LOADS

D - effect of dead load.

L - effect of design live load including impact, where applicable.

W - effect of wind load, or earthquake load, or traction forces.

V_c - shear carried by concrete.

104.4 STRESSES AND STRAINS

E_c - flexural modulus of elasticity of concrete.

E_s - modulus of elasticity of prestressing steel.

f'_c - compressive strength of concrete at 28 days.

f'_{ci} - compressive strength of concrete at time of initial prestress.

f_{cp} - permissible compressive concrete stress on bearing area under anchor plate of posttensioning steel.

f'_s - ultimate strength of prestressing steel.

f_{se} - effective steel prestress after losses.

f_{si} - initial stress in prestressing steel after seating of the anchorage.

f_{su} - stress in prestressing steel at ultimate load.

f_{sy} - nominal yield point stress of prestressing steel.

f'_t - flexural tensile strength of concrete; modulus of rupture.

f'_y - yield point stress of conventional reinforcing steel.

k_2 - ratio of distance between extreme compressive fiber and center of compression to depth to neutral axis.

k_1k_3 — ratio of average compressive concrete stress to cylinder strength, f'_c.

n - ratio of E_s/Ec.

u_d - strain in concrete due to creep.

u_e - strain in concrete due to elastic shortening.

u_s - strain in concrete due to shrinkage.

v - shearing stress.

δ_1 - ratio of loss in steel stress due to relaxation of prestressing steel.

δ_2 - ratio of loss in steel stress due to friction during prestressing.

104.5 FRICTION DURING PRESTRESSING

e - base of Naperian Logarithms.

K - friction wobble coefficient per foot of prestressing steel.

T_o - steel stress at jacking end.

T_x - steel stress at any point x.

μ - friction curvature coefficient.

α - total angular change of prestressing steel profile in radians from jacking end to point x.

L - length of prestressing steel element from jacking end to point x.

CHAPTER 2 - DESIGN

SECTION 201 - GENERAL CONSIDERATIONS

201.1 PURPOSE

The purpose of design is to define a structure that can be constructed economically, that will perform satisfactorily under service conditions, and will have an adequate ultimate load capacity.

201.2 MODE OF FAILURE

Ultimate strength should be governed preferably by elongation of the prestressing steel rather than by shear, bond, or concrete compression.

201.3 DESIGN THEORY

The elastic theory should be used at design loads with internal stresses limited to recommended values. The ultimate strength theory also should be applied to insure that ultimate capacity provides the recommended load factors.

SECTION 202 - SPECIAL CONSIDERATIONS

202.1 LOADING CONDITIONS

Consideration should be given to all critical loading conditions in design including those that occur during fabrication, handling, transportation, erection and construction.

202.2 DEFLECTIONS

Camber and deflection may be design limitations and should be investigated for both short and long time effects.

202.3 LENGTH CHANGES

Length changes of concrete due to prestress and other causes should be investigated for both short and long time effects.

202.4 REVERSAL OF LOADING EFFECTS

Where reversal of moment or shear may occur it should be considered in the design.

202.5 BUCKLING

General buckling due to prestressing can occur only over the length between points of contact of the prestressing steel with the concrete.

General buckling of an entire member or local buckling of thin webs and flanges under external loads may occur in prestressed concrete as in members made of other materials and should be provided for in design.

SECTION 203 - ASSUMPTIONS

203.1 BASIC ASSUMPTIONS

The following assumptions may be made for design purposes:

a. Strains vary linearly over the depth of the member throughout the entire load range.

b. Before cracking, stress is linearly proportional to strain.

c. After cracking, tension in the concrete is neglected.

203.2 MODULUS OF ELASTICITY

When accurate values for modulus of elasticity are not available, the following values may be used as a guide:

a. Flexural modulus of elasticity of concrete, E_c, in psi, may be assumed to be 1,800,000 plus 500 times the cylinder strength at the age considered. Actual values may vary as much as 25 per cent from those given by the foregoing expression. This expression is not applicable to lightweight concrete, for which E_c should be determined by test.

b. Modulus of elasticity in psi, of steel may be assumed to be 29,000,000 for cold drawn wire, 27,000,000 for 7 wire strand, 25,000,000 for strand with more than 7 wires, and 27,000,000 for alloy steel bars.

203.3 DEFLECTIONS

Deflection or camber under short time loading may be computed using values of E_c obtained as described in Sec. 203.2.a.

Deflection associated with dead load, prestress, and live loads sustained for a long time may be computed on the assumption that the corresponding concrete strains are increased as a result of creep. The increase in strain may vary from 100 per cent of the elastic strain in very humid atmosphere to 300 per cent of the elastic strain in very dry atmosphere. These values may not pertain to concrete made with lightweight aggregates.

SECTION 204 - LOADING STAGES

204.1 LOADING

Loading stages listed in the following sections should be investigated. No attempt is made to list all significant loading stages that may occur. Stages listed are those that normally affect the design.

204.2 INITIAL PRESTRESS

Prestressing forces are applied in prearranged sequence and sometimes in stages. If prestressing forces are not counteracted by the effect of the dead load of the member, or if the stressing operation is accompanied by temporary eccentricities, concrete stresses should be investigated.

204.3 INITIAL PRESTRESS PLUS DEAD LOAD OF MEMBER

For determination of concrete stresses at this stage, losses in prestress are those which occur during and immediately after transfer of prestress.

204.4 TRANSPORTATION AND ERECTION

Support conditions for precast members during transportation and erection may differ from those during service loads. Handling stresses should be included together with prestress and dead load. Losses in initial prestress up to time of handling should be considered.

204.5 DESIGN LOAD

This stage includes stress due to effective prestress after losses, dead loads and maximum specified live load.

204.6 CRACKING LOAD

Complete freedom from cracking may or may not be necessary at any particular loading stage. Type and function of the structure and type, frequency and magnitude of live loads should be ccnsidered.

204.7 TEMPORARY OVERLOAD

This stage refers to any large live load in excess of design load, which is of short-time duration and expected to occur infrequently during life of structure. For such a load, stresses may exceed those recommended for design load but elastic recovery must be assured.

204.8 ULTIMATE LOAD

Ultimate load is that load which applied statically in a single application causes failure. Such a large load would never intentionally be placed on the structure, but it is used as a measure of safety. In statically determinate structures, failure will occur at a single cross-section. In statically indeterminate structures, the load which causes moment in one section to reach its ultimate value may not be sufficient to cause failure of the structure because of moment redistribution. Since it is not always possible to predict that full redistribution will take place in accordance with limit design, it is suggested for the time being that moments be determined by elastic analysis.

SECTION 205 - LOAD FACTORS

205.1 GENERAL

A load factor is a multiple of the design loads used to insure safety of the structure.

205.2 CRACKING LOAD FACTORS

If cracking of concrete is undesirable, load factors for cracking load should be chosen to reflect the greatest load that can be expected during life of structure.

Formation of a crack under temporary overload may not be objectionable. If reopening of such a crack under subsequent design load is objectionable it may be avoided by proper choice of concrete stress permitted for cracking load.

205.3 ULTIMATE LOAD FACTORS

The ultimate load capacity should be computed since stresses are not linearly proportional to external forces and moments throughout the entire load range. For the present, it is recommended that moments, shears, and thrusts produced by external loads and prestressing forces be investigated by elastic analysis.

The load factors recommended are believed to be consistent with current viewpoints. It may be desirable to modify or expand the load factor formulas to fit special conditions that may occur in unusual structures, extremely long spans, or for unique loadings. Deviations from the recommended values should be substantiated by suitable investigations

205.3.1 <u>Buildings</u>

For the present, to correlate prestressed concrete with reinforced concrete practice in current use, the committee recommends that ultimate load capacity be investigated to insure meeting the following requirements:

$$1.2\ D + 2.4\ L$$

or $$1.8\ (D + L)$$

or $$1.2\ D + 2.4\ L + 0.6\ W$$

or $$1.2\ D + 0.6\ L + 2.4\ W$$

or $$1.8\ (D + L + 1/2\ W)$$

or $$1.8\ (D + 1/2\ L + W)$$

whichever is greater.

205.3.2 <u>Highway Bridges</u>

The following load factors for highway bridges are recommended by a sub-group appointed by American Association of State Highway Officials, Committee on Bridges and Structures.*

$$1.5\ D + 2.5\ L$$

The committee is not prepared at this time to make recommendations for load factors involving the effect of lateral loads on bridges.

* These load factors are considered adequate for spans of moderate length, simply supported. For exceptionally long spans and for continuous members, special investigation to consider a possible increase in load factors is recommended.

205.3.3 Railway Bridges

Ultimate load factors for railway bridges are currently being studied by the American Railway Engineering Association. The committee is not prepared to recommend such factors at this time.

SECTION 206 - REPETITIVE LOADS

206.1 GENERAL

Ultimate strength of concrete or steel subjected to repetitive loading may be less than static strength because of the phenomenon of fatigue. Full importance of fatigue in prestressed concrete members has not yet been determined. Fatigue failure may occur in concrete, steel, anchorages, splices or bond.

206.2 CONCRETE

Fatigue strength of concrete in both tension and compression depends on magnitude of stress, range of stress variation, and number of loading cycles. Since high stresses and stress ranges are common, fatigue should be considered when repetition of loading cycles may occur.

Fatigue failure is unlikely if the allowable stresses of Section 207.3.2 are not exceeded and there is no reversal of stress. If a large number of overloads are anticipated a reduction in the safety factor may occur.

206.3 PRESTRESSING STEEL

Fatigue strength of prestressing steel depends on magnitude and range of stress, and number of cycles of loading. Minimum stress is the effective prestress. Maximum stress and range of stress depend on magnitude of live loads or overloads that may be repeated. Range of stress under service loads will usually be small unless concrete is cracked. Cracking may occur if tension is permitted in concrete. Fatigue failure of steel should be considered in such cases, especially when a high percentage of ultimate strength is used for prestress.

Devices for splicing steel may contain strain concentrations that lower fatigue strength. Consideration should be given to fatigue whenever splices are used.

206.4 ANCHORAGES

If steel is fully bonded, no difficulty should be expected in the anchorage or end bearing as the result of repetitive loads. With unbonded steel, fluctuations in stress due to repeated service loads or overloads are transmitted directly to anchorages and fatigue strength of the anchorage will require special consideration.

206.5 BOND

Failure of bond under repetitive loading is unlikely unless the member is cracked under design loads or a significant number of

repetitions of overload. High bond stresses adjacent to cracks may be a source of progressive failure under repeated loads.

206.6 SHEAR AND DIAGONAL TENSION

Since inclined cracks may form under repetitive loading at appreciably smaller stresses than under static loading, web reinforcement should always be provided in members subjected to repetitive loading.

206.7 DESIGN RECOMMENDATIONS

Fatigue should not result in a reduction of strength if the following recommendations are observed. When the recommendations cannot be followed, fatigue strength of all elements comprising the prestressed member should be considered.

a. Flexural compressive concrete stress should not exceed $0.4f'_c$ under either design load or an overload that may be repeated a large number of times.

b. Tension should not be permitted in concrete at the critical cross-section under either design load or overloads that may be repeated a large number of times.

c. Reversal of stress should not occur under repeated loads.

d. Prestressing steel should be bonded.

e. Web reinforcement should be provided.

SECTION 207 - ALLOWABLE STEEL AND CONCRETE STRESSES

207.1 PRESTRESSING STEEL

207.1.1 Temporary Stresses

Under normal design loads stress in prestressing steel will almost always be less than stress at initial prestress. Stress at the anchorage immediately after seating has been effected should not exceed $0.70f'_s$ for material having stress-strain properties defined in Chapter 3. Overstressing for a short period of time to $0.80f'_s$ may be permitted provided the stress, after seating of anchorage occurs, does not exceed $0.70f'_s$.

207.1.2 Stress at Design Loads

Effective steel stress after losses described in Section 208 should not exceed:

$$0.60f'_s \text{ or } 0.80f_{sy}$$

whichever is smaller.

207.2 NON-PRESTRESSED REINFORCEMENT

Non-prestressed reinforcement provided to resist tension in conformance with requirements of Section 207.3.1.b.2 may be assumed stressed to 20,000 psi.

207.3 CONCRETE

207.3.1 Temporary Stresses

Concrete stress in psi before losses due to creep and shrinkage should not exceed the following:

a. Compression
   For pretensioned members . . . . $0.60f'_{ci}$
   For posttensioned members . . . . $0.55f'_{ci}$

b. Tension
   1. For members without non-prestressed reinforcement:
      Single element . . . . $3\sqrt{f'_{ci}}$
      Segmental element . . . . zero

   2. For members with non-prestressed reinforcement provided to resist the tensile force in the concrete, computed on the basis of an uncracked section:
      Single element . . . . $6\sqrt{f'_{ci}}$
      Segmental element . . . . $3\sqrt{f'_{ci}}$

207.3.2 Stresses at Design Loads

After full prestress losses, stresses in psi should not exceed the following:

a. Compression
   1. Single element
      a. bridge members . . . . $0.40f'_c$
      b. building members . . . . $0.45f'_c$

   2. Segmental elements
      a. bridge members . . . . $0.40f'_c$
      b. building members . . . . $0.45f'_c$

b. Flexural tension in the precompressed tensile zone
   1. Single element
      a. bridge members . . . . zero
      b. pretensioned building elements not exposed to weather or corrosive atmosphere . . . $6\sqrt{f'_c}$
      c. posttensioned bonded elements not exposed to weather or corrosive atmosphere . . . $3\sqrt{f'_c}$

   2. Segmental elements
      a. bridge members . . . . zero
      b. building members . . . . zero

Allowable flexural tension of $6\sqrt{f'_c}$ in Section 207.3.2.b.1.b may be exceeded provided it is shown by tests that the structure will behave properly under service conditions and meet any necessary requirement for cracking load or temporary overload.

207.3.3 Stress at Cracking Load

Flexural tensile strength (modulus of rupture) should preferably be determined by test. When test data are not available the ultimate

flexural tensile stress in psi may be assumed as:

$$f'_t = 7.5\sqrt{f'_c}$$

For lightweight concrete, f'_t should be determined by tests.

207.3.4 Anchorage Bearing Stresses

The maximum allowable stress at posttensioning anchorage in end blocks adequately reinforced in conformance with Section 214.4 may be assumed as:

$$f_{cp} = 0.6\, f'_{ci} \sqrt[3]{A_c/A_b}$$

where A_b = bearing area of the anchor plate.

A_c = maximum area of portion of the member that is geometrically similar to and concentric with the area of bearing plate.

The allowable value of f_{cp} should not exceed f'_{ci}.

SECTION 208 - LOSS OF PRESTRESS

208.1 INTRODUCTION

Initial prestress is that stress in steel which exists immediately after seating of anchorage. Stress diminishes with time and finally reaches a stable condition of effective prestress assumed to be permanent.

208.2 SOURCES OF PRESTRESS LOSS

208.2.1 Friction Loss in Posttensioned Steel

If posttensioned steel is draped, or irregularities exist in alignment of ducts, steel stress will be less within the member than at the jack because of friction between prestressing steel and duct. Magnitude of this friction should be estimated for design and verified during stressing operation.

Friction loss may be estimated from an analysis of forces exerted by prestressing steel on duct. One method for determination of friction loss at any point is given below.

$$T_o = T_x e^{(KL + \mu\alpha)}$$

where T_o = steel stress at jacking end

T_x = steel stress at point x

e = base of Naperian Logarithms

K = friction wobble coefficient per foot of prestressing steel

L = length of prestressing steel element from jacking end to point x in feet

μ = friction curvature coefficient

α = total angular change of prestressing steel element in radians from jack to point x.

For small values of KL and $\mu\alpha$ the following formula may be used:

$$T_o = T_x (1 + KL + \mu\alpha)$$

The following values of K and μ are typical and may be used as a guide. They may vary appreciably with duct material and method of construction. Values of K and μ used in design should be indicated on the plans for guidance in selection of materials and methods that will produce results approaching the assumed values.

TYPE STEEL	TYPE OF DUCT OR SHEATH	USUAL RANGE OF OBSERVED VALUES		SUGGESTED DESIGN VALUES	
		K	μ	K	μ
Wire Cables	bright metal sheathing	0.0005-0.0030	.15-.35	0.0020	0.30
	galv. metal sheathing			0.0015	0.25
	greased or asphalt coated and wrapped	0.0030	.25-.35	0.0020	0.30
High Strength Bars	bright metal sheathing	0.0001-0.0005	.08-.30	0.0003	0.20
	galv. metal sheathing			0.0002	0.15
Galv. Strand	bright metal sheathing	0.0005-0.0020	.15-.30	0.0015	0.25
	galv. metal sheathing			0.0010	0.20

Workmanship in placing, supporting, tying, and fabricating prestressing elements and ducts influences the magnitude of wobble factor, K. The larger the sheath or duct in relation to the size of prestressing steel element, the smaller K will be. With normal placing tolerances wobble effect may be neglected if sheath is one inch greater in diameter than prestressing steel element.

Effect of overestimating friction loss should be considered since excessive prestress may cause undesirable permanent stress conditions. Underestimating friction loss may result in an error in computed cracking load and deflection.

208.2.2 Elastic Shortening of Concrete

Loss of prestress caused by elastic shortening of the concrete occurs in prestressed concrete members. This loss equals $n(\Delta f_c)$. For pretensioned concrete, Δf_c is the concrete stress at the center of gravity of the prestressing steel for which the losses are being computed. For posttensioned concrete where the steel elements may not be tensioned simultaneously, Δf_c is the average concrete stress along one prestressing element from end to end of the beam caused by subsequent posttensioning of adjacent elements.

208.2.3 Shrinkage of Concrete

Shrinkage depends on many variables. Unit shrinkage strain may vary from near 0 to 0.0005. A value between 0.0002 and 0.0003 is commonly used for calculation of prestress loss. Shrinkage loss may be greater in pretensioned members where the prestress is transferred to the concrete at an earlier age than is usual for posttensioned members. Shrinkage of lightweight concrete may be greater than the values obtained with the above factors.

208.2.4 Creep of Concrete

Creep is the time-dependent strain of concrete caused by stress. For pretensioned and posttensioned bonded members, concrete stress is taken at center of gravity of prestressing steel under effect of prestress and permanent loads (normal conditions of unloaded structure).

In posttensioned unbonded members, stress is the average concrete stress along the profile of center of gravity of prestressing steel under the effect of prestress and permanent loads. Additional strain due to creep may be assumed to vary from 100 per cent of elastic strain for concrete in very humid atmosphere to 300 per cent of elastic strain in very dry atmosphere.

Creep of some lightweight concretes may be greater than indicated above.

208.2.5 Relaxation of Steel Stress

Loss of stress due to relaxation of prestressing steel should be provided for in design in accordance with test data furnished by the steel manufacturer. Loss due to relaxation depends primarily on properties of the steel and initial prestress. This loss is generally assumed in the range of two to eight per cent of initial steel stress.

208.3 ALTERNATE PROCEDURES FOR ESTIMATING PRESTRESS LOSSES

Two methods are suggested for estimating prestress losses. Method 1 should be used when individual losses may be predicted with reasonable accuracy. Method 2 applies when specific loss data are lacking.

The ultimate strength is not significantly affected by the magnitude of steel stress loss. An error in choosing the loss is reflected in the cracking load and amount of camber.

208.3.1 Method 1

The total stress loss in prestressing steel:

$$\Delta f_s = (u_s + u_e + u_d)\,E_s + \delta_1 f_{si} + \delta_2 f_{si}$$

208.3.2 Method 2

Loss in steel stress not including friction loss may be assumed as follows:

Pretensioning	35,000 psi
Posttensioning	25,000 psi

For camber calculations these values may be excessive.

208.4 LIGHTWEIGHT CONCRETE

Losses due to concrete shrinkage, elastic shortening and creep should be based on results of tests made with the lightweight aggregate to be used.

SECTION 209 - FLEXURE

209.1 STRESSES DUE TO DEAD, LIVE, AND IMPACT LOADS

Prestressed concrete members may be assumed to function as uncracked members subjected to combined axial and bending forces provided stresses do not exceed those given in Section 207.

In calculations of section properties prior to grouting, areas of the open ducts should be deducted unless relatively small. The transformed area of bonded reinforcement may be included in pretensioned members and posttensioned members after grouting.

For calculation of stress due to prestress in T-beams no definite recommendations are made at this time, but attention should be given to the possibility that the entire available flange width may be included in calculation of section properties.

209.2 ULTIMATE FLEXURAL STRENGTH

209.2.1 General Method

(a) Rectangular Sections

For rectangular sections or flanged sections in which the neutral axis lies within the flange, ultimate flexural strength may be expressed as:

$$M_u = A_s f_{su} d\left(1 - \frac{k_2}{k_1 k_3} \frac{p f_{su}}{f'_c}\right) \qquad \text{(A)}$$

where f_{su} = average stress in prestressing reinforcement at ultimate load

d = depth to centroid of force

k_2 = ratio of distance between extreme compressive fiber and center of compression to the depth to neutral axis

$k_1 k_3$ = ratio of average compressive concrete stress to the cylinder strength, f'_c.

The results of numerous tests have shown that the factor

k_2/k_1k_3 may be taken equal to 0.6 for members and materials considered in this report. Determination of the value of f_{su} requires knowledge of the stress-strain characteristics of the prestressing steel, effective prestress and crushing strain of the concrete. Assumptions must be made regarding the relation between steel and concrete strains. These assumptions will be different for bonded and unbonded construction.

The ultimate moment may be computed from Eq. (A) whenever sufficient information is available for the determination of f_{su}. The approximate method of Section 209.2.2 may be used if the required conditions are satisfied.

(b) Flanged Sections

If a flange thickness is less than $1.4dpf_{su}/f'_c$, the neutral axis will usually fall outside the flange and the following approximate expression for ultimate moment should be used:

$$M_u = A_{sr}f_{su}d(1 - 0.6\frac{A_{sr}f_{su}}{b'df'_c}) + 0.85\, f'_c\,(b - b')t\,(d - 0.5t) \qquad \text{(B)}$$

where $A_{sr} = A_s - A_{sf}$ = the steel area required to develop the ultimate compressive strength of the web of a flanged section.

$A_{sf} = 0.85f'_c\,(b-b')t/f_{su}$ = steel area required to develop the ultimate compressive strength of the overhanging portions of the flange.

t = average thickness of flange

The expressions for f_{su} given in Section 209.2.2 may be used if the required conditions are satisfied.

209.2.2 Approximate Method

The following approximate expressions for f_{su} may be used in equations (A) and (B) of Section 209.2.1 provided the following conditions are satisfied:

1. The stress-strain properties of the prestressing steel are reasonably similar to those described in Section 304.
2. The effective prestress after losses is not less than $0.5f'_s$.

(a) Bonded Members

$$f_{su} = f'_s\,(1 - 0.5\frac{pf'_s}{f'_c})$$

(b) Unbonded Members

Ultimate flexural strength in unbonded members generally occurs at lower values of steel stress than in bonded members. Wide variations between stress levels reported by different investigators reflect the fact that several factors influence the stress

developed by unbonded steel at ultimate moment. These factors include: magnitude of effective prestress, profile of the prestressing steel, shape of the bending moment diagram, length/depth ratio of the member, magnitude of the friction coefficient between the prestressing steel and duct, and amount of bonded non-prestressed supplementary steel.

Unless the proper value of f_{su} is known from tests of members closely approximating proposed construction with respect to the several factors listed in the preceding paragraph, it is recommended that:

$$f_{su} = f_{se} + 15{,}000$$

209.2.3 Maximum Steel Percentage

To avoid approaching the condition of over-reinforced beams for which the ultimate flexural strength becomes dependent on the concrete strength, the ratio of prestressing steel preferably should be such that pf_{su}/f'_c for rectangular sections, and $A_{sr}f_{su}/b'df'_c$ for flanged sections are not more than 0.30.

If a steel ratio in excess of this amount is used, the ultimate flexural moment shall be taken as not greater than the following values when either the general or approximate method of calculation is used.

(a) Rectangular Sections

$$M_u = 0.25\, f'_c bd^2$$

(b) Flanged Sections

If the flange thickness is less than $1.4\, dpf_{su}/f'_c$ the neutral axis will usually fall outside the flange and the following formula is recommended.

$$M_u = 0.25bd^2 f'_c + 0.85 f'_c\, (b-b')t(d-0.5t)$$

209.2.4 Non-prestressed Reinforcement in Conjunction with Prestressing Steel

209.2.4.1 Conventional Reinforcement

Non-prestressed conventional reinforcement may be considered to contribute to the tensile force in the beam at ultimate moment an amount equal to its area times its yield point provided that

$$\frac{pf_{su}}{f'_c} + \frac{p'f'_y}{f'_c} \text{ does not exceed } 0.3$$

where f'_y = yield point of conventional reinforcement.

p' = ratio of conventional reinforcement.

209.2.4.2 High Tensile Strength Reinforcement

If untensioned prestressing steel or other high tensile strength

reinforcement is used in conjunction with prestressed reinforcement, the ultimate moment should be calculated by means of the general method of Section 209.2.1.

SECTION 210 - SHEAR

210.1 GENERAL

210.1.1 Ultimate Strength

It is essential that shear failure should not occur before ultimate flexural strength required in Section 209.2 is developed. If this condition is satisfied, it is unnecessary to investigate shear or principal tensile stresses at design loads.

210.1.2 Inclined Cracking

Formation of inclined cracks precedes failure in shear. They are caused by inclined principal tensile stresses that are the resultant of shearing stresses and normal bending stresses. Compressive prestress reduces the principal tensile stress thereby increasing the load necessary to cause inclined cracks. The use of thin webs will increase inclined stresses.

210.1.3 Conditions for Shear Failure

The resistance to formation of inclined cracks is greater with larger prestress and increasing web thickness. The significance of inclined cracks is less with low ultimate flexural strength caused by low ratio of reinforcement. Their significance is also less with low shear/moment ratios. If inclined cracks occur in an unreinforced web, sudden failure by shear is almost certain. If the web is adequately reinforced, ultimate flexural strength can be developed.

210.2 WEB REINFORCEMENT

210.2.1 Critical Percentage of Tensile Steel

Experimental data, although limited, indicate that inclined tension cracks will not form and web reinforcement will not be required if the following condition is satisfied

$$\frac{pf'_s}{f'_c} \leqq 0.3 \frac{f_{se}}{f'_s} \frac{b'}{b}$$

where b' = thickness of web

b = width of flange corresponding to that used in computing p.

This expression may be conservative for members having span/depth ratios greater than about 15 or for uniformly loaded members. In such cases, web reinforcement may not be required even though

the percentage index, pf'_s/f'_c, exceeds that given in the above expression. The omission of web reinforcement in such members may be allowed when justified by tests.

210.2.2 Design of Web Reinforcement

The amount of web reinforcement necessary to develop required ultimate flexural capacity is a function of the difference between inclined cracking load and ultimate load in flexure. This difference varies rather widely as a function of prestress force, web thickness, amount of tensile reinforcement and shear/moment ratio but is usually smaller for prestressed concrete than for conventional reinforced concrete. Current design procedures for web reinforcement in reinforced concrete are conservative for prestressed concrete.

Available test data indicate that the following expression for area of web reinforcement, with its factor of 1/2, will give reasonably conservative results for prestressed members of usual dimensions and properties. Since the formula does not involve the prestress force it may not be conservative for very low prestress or where only a portion of the reinforcement is stressed. For such cases it may be necessary to increase the factor of 1/2 as the member approaches the condition of conventionally reinforced concrete.

$$A_v = \frac{1}{2}\,\frac{(V_u - V_c)s}{f'_y jd}$$

where A_v = area of web reinforcement at spacing, s placed perpendicular to the axis of the member

V_u = shear due to specified ultimate load and effect of prestressing

V_c = $0.06f'_c\,b'jd$ but not more than 180 b'jd

s = longitudinal spacing of web reinforcement

f'_y = yield strength of web reinforcement

210.2.3 Minimum Quantity of Web Reinforcement

Because of the nature and limited knowledge of shear failures, it is suggested that some web reinforcement be provided even though the criterion of Section 210.2.1 is satisfied.

Where the web reinforcement is designed by Section 210.2.2, the minimum amount of web reinforcement should be $A_v = 0.0025b's$. This requirement may be excessive for members with unusually thick webs and the amount of web reinforcement may be reduced if tests demonstrate that the member can develop its required flexural capacity.

Heavily loaded members with thin webs and relatively small span/depth ratios, such as highway bridge girders and crane girders should have web reinforcement. (See Section 206.6)

210.2.4 Spacing of Web Reinforcement

The spacing of web reinforcement should not exceed 3/4 the depth of the member. In members with relatively thin webs, spacing should preferably not exceed the clear height of the web.

210.2.5 Critical Sections for Shear

Because formation of inclined cracks reduces flexural capacity the critical sections for shear will usually not be near the ends of the span where the shear is a maximum but at some point away from the ends in a region of high moment.

For the design of web reinforcement in simply supported members carrying moving loads, it is recommended that shear be investigated only within the middle half of the span length. The web reinforcement required at the quarter-points should then be used throughout the outer quarters of the span.

For simply supported members carrying only uniformly distributed load, the maximum web reinforcement may be taken as that required at a distance from the support equal to the depth of member. This amount of web reinforcement should be provided from this point to the end of member. In the middle third of the span length, the amount of web reinforcement provided should not be less than that required at third-points of the span.

SECTION 211 - BOND AND ANCHORAGE

211.1 PRETENSIONING

211.1.1 Prestress Transfer Bond

Bond between the pretensioned steel and concrete is necessary to establish a prestress in the concrete. The transfer of force from the steel to the concrete takes place in a finite length in the end region of a member and the function of the resulting bond, termed "prestress transfer bond," is anchorage of prestressing steel. Prestressing force varies from near zero at the end to a maximum value some distance from the end.

Transfer length will generally be of minor significance in long members, but it should be considered for short members or those in which the loading conditions may cause cracking in or near the region of prestress transfer.

211.1.2 Flexural Bond

Flexural bond is the bond stress developed as a consequence of flexure. Bond stress at design loads in uncracked members is usually not critical since the increase in steel stress resulting from flexure is usually not significant. If cracking is anticipated under design loads, bond stress should be given special consideration.

211.1.3 Significance of Bond Stress at Ultimate Load

Bond failure should not occur prior to the development of the required ultimate flexural capacity.

For span lengths usually associated with prestressed concrete, bond failure is not a significant design factor. Bond adequacy in extremely short members should be investigated by test.

The factors affecting bond are concrete strength, perimeter shape, area and surface condition of prestressing steel, stress in the steel at ultimate strength, length of transfer zone and superimposed load pattern.

SECTION 212 - COMPOSITE CONSTRUCTION

212.1 INTRODUCTION

Prestressed concrete structures of composite construction are comprised of prestressed concrete elements and plain or conventionally reinforced concrete elements interconnected in such a manner that the two components function as an integral unit. The prestressed elements may be pretensioned or posttensioned and may be precast or cast in place. The plain or reinforced concrete elements are usually cast in place.

212.2 INTERACTION

212.2.1 Shear Connection

To insure integral action of a composite structure at all loads, a connection should be provided between the component elements of the structure capable of performing two functions:

(1) To transfer shear without slip along the contact surfaces, and
(2) To prevent separation of the elements in a direction perpendicular to the contact surfaces.

212.2.2 Transfer of Shear

Slip may be prevented and shear transferred along the contact surfaces either by bond or by shear keys. It should be assumed that the entire shear is transferred either by bond or by shear keys.

212.2.3 Anchorage Against Separation

Mechanical anchorage in the form of vertical ties should be provided to prevent separation of the component elements in the direction perpendicular to the contact surfaces. Web reinforcement or steel dowels adequately embedded on each side of the contact surface will provide satisfactory mechanical anchorage.

212.3 DESIGN OF SHEAR CONNECTION

212.3.1 Loading Stage

The shear connection should be designed for ultimate load.

212.3.2 Magnitude and Transfer of Ultimate Shear

The shear at any point along the contact surface may be computed by the usual method as $v = \frac{V_u Q}{I}$. If the bond capacity is less than the computed shear, full width shear keys should be provided throughout the length of the member. Keys should be proportioned according to concrete strength of each component of the composite member.*

212.3.3 Capacity of Bond

The following values are suggested for ultimate bond resistance of the contact surfaces.

When minimum steel tie requirements of Section 212.3.4 are followed	75 psi
When minimum steel tie requirements of Section 212.3.4 are followed and the contact surface of the precast element is artificially roughened	150 psi
When additional steel ties in excess of the requirements of Section 212.3.4 are used and the contact surface of the precast element is artifically roughened	225 psi

212.3.4 Vertical Ties

In the absence of experimental information on the capacity of vertical ties it is recommended that all web reinforcement be extended into the cast-in-place concrete.

Spacing of vertical ties should not exceed 4 times the minimum thickness of the composite elements, or 24 in. whichever is less. The total area of vertical ties should not be less than that provided by two No. 3 bars spaced at 12 in.

For light pretensioned members such as those used for building floors not subjected to repetitive loads the above minimum requirements may be too severe. The Committee is not prepared to recommend an amount or spacing of steel for this type member.

212.4 DESIGN OF COMPOSITE STRUCTURES

212.4.1 Design of Composite Section

Physical properties of the composite section should be computed on the assumption of complete interaction between component elements. For structures composed of concretes of different qualities, the area of one of the component elements should be transformed in accordance with the ratio of the two moduli of elasticity.

* Lack of experimental data makes the committee hesitate to recommend a shear stress at the root of a key. Indications are that for keys on bridge girders in current use shear stress at the root of a key as high as $0.3f'_c$ would sometimes be required to transmit ultimate shear force.

212.4.2 Beam and Slab Construction

If the structure is composed of beams with a cast-in-place slab placed on top of the beams, effective slab width should be computed in the same manner as for integral T-beams.

212.4.3 Allowable Stress with Different Concrete Strengths

In structures composed of elements with different concrete strengths, the allowable stresses should be governed by strength of the portion under consideration.

212.4.4 Superposition of Stress

Stresses may be superposed in design calculations that involve elastic stresses. Superposition of stresses should not be used in computing ultimate strength since inelastic action of the material is involved.

212.4.5 Stress after Structure Becomes Integral

The properties of the composite cross-section should be used in computing stresses due to loads applied after the structure becomes integral.

212.4.6 Shrinkage Stresses

In structures with a cast-in-place slab supported by precast beams, the differential shrinkage tends to cause tensile stresses in the slab and in the bottom of precast beams. Stresses due to differential shrinkage are important only insofar as they affect cracking load. When cracking load is significant, such stresses should be added to the effects of loads.

212.4.7 Ultimate Strength

Ultimate strength of a composite section should be computed in the same manner as ultimate strength of an integral member of the same shape.

SECTION 213 - CONTINUITY

213.1 DETERMINATION OF MOMENTS, SHEARS, AND THRUSTS

Moments, shears and thrusts produced by external loads and prestressing force should be determined by elastic analysis. Effects of axial deformation should be considered. Determination of effects produced by the prestressing forces should take into account the restraint of attached structural elements and supports.

213.2 STRESSES

Allowable stresses are those recommended in Section 207.

213.2.1 Prestress

When prestressing is to be applied in more than one stage, the internal stresses should be investigated at each stage.

213.3 FRICTIONAL LOSSES

Frictional losses in continuous posttensioned steel may be more significant than in simply supported members.

213.4 ULTIMATE STRENGTH

The ultimate strength of a continuous member should be evaluated not only at points of maximum moment, but also at intermediate points. In applying ultimate load factors where dead load causes effects opposite to those of live load, consideration should be given to load factor combinations in which dead load factor may equal one. It is recommended that moment redistribution not be considered in design at the present time.

SECTION 214 - END BLOCKS

214.1 PURPOSE

An enlarged end section, called an end block, may be required to transmit concentrated prestressing forces in a shaped member from the anchorage area to the basic cross section.

End blocks may be required to provide sufficient area for bearing of anchorages in posttensioned design. They may be needed to transmit vertical and lateral forces to supports and to facilitate end detailing.

214.2 REQUIREMENTS

In pretensioned members with large concentrated eccentric prestressing elements, end blocks should be used. For lightly pretensioned members, or members of approximately rectangular shape, end blocks may be omitted. However, reinforcement should always be provided in the anchorage zone.

In posttensioned, shaped members, end blocks should be provided.

214.3 PROPORTIONING

End blocks are usually proportioned by experience. Depending on the degree of concentration and eccentricity of the prestressing force at the end surface, the length of the end block should be from one half the depth of the member to the full depth. In general, shallow members should have an end block length equal to the depth, and deep beams should have an end block length equal to three quarters of the depth. Length of an end block can be considered as the distance from beginning of anchorage area to the point where the end block intersects the narrowest width of member.

214.4 REINFORCEMENT

Reinforcing is necessary to resist tensile bursting and spalling forces induced by the concentrated loads of the prestressing steel. A reinforcing grid with both vertical and horizontal steel in the plane of the cross-section should be provided directly beneath

anchorages to resist spalling forces. Closely spaced reinforcement should be placed both vertically and horizontally throughout the length of the end block to resist tensile forces.

SECTION 215 - FIRE RESISTANCE

215.1 GENERAL

The fire resistance of both prestressed concrete and reinforced concrete is subject to the same general limitations. One is the rate of heat transmission through the concrete from the surface exposed to fire to the unexposed surface. The other is the reduction of steel strength at the temperatures induced in the steel during the test. Either limitation may govern.

215.2 HEAT TRANSMISSION

Since the rate of heat transmission through prestressed concrete is similar to that of reinforced concrete of the same composition, the critical dimensions to control temperature rise at the unexposed surface will be the same in prestressed or reinforced concrete members.

215.3 LOAD CARRYING CAPACITY

The ability of the structure to carry required loads during fire test depends largely on thickness of cover over prestressing steel. The following minimum thicknesses of concrete cover on prestressing steel and end anchorages are recommended for various fire ratings.

Hour rating	1 hr.	2 hr.	3 hr.	4 hr.
Minimum concrete cover	1-1/2 in.	2-1/2 in.	3 in.	4 in.

Data now available are insufficient to make recommendations for such factors as shape of cross-section, type and arrangement of prestressing steel. The cover thicknesses recommended are believed to be conservative.

SECTION 216 - COVER AND SPACING OF PRESTRESSING STEEL

216.1 COVER

The following minimum clear concrete covers are recommended for prestressing steel, ducts and non-prestressed steel.

	Minimum Concrete Cover
Concrete surfaces exposed to weather	1-1/2 in.
Concrete surfaces in contact with ground	2 in.
Beams and girders not exposed to weather	
Prestressing steel, and main reinforcing steel	1-1/2 in.
Stirrups and ties	1 in.
Slabs and joists not exposed to weather	3/4 in.

216.2 SPACING AT ENDS

216.2.1 Spacing of Pretensioning Steel

Minimum horizontal or vertical clear spacing between pretensioning steel elements at ends of members should be three times the diameter of the steel or 1-1/3 times the maximum size of coarse aggregate, whichever is greater.

216.2.2 Spacing of Posttensioning Ducts

The clear space between conduits at the ends should be a minimum of 1-1/2 in. or 1-1/2 times the maximum size of coarse aggregate, whichever is greater.

216.2.3 Dimensions of Posttensioning Ducts

When steel is placed inside conduits which are to be filled with cement grout, such conduits should have a minimum inside diameter 1/4 in. larger than the diameter of the prestressing steel.

216.3 DRAPED PRESTRESSING STEEL

When prestressing steel is placed in a curved or deflected position, steel or conduits may be bundled together in the middle third of the span length provided the minimum spacing recommended in Section 216.2.1 and 216.2.2 is maintained for a minimum distance of 3 ft. at each end of member. The committee is not prepared to suggest limits for the number of conduits or prestressing steel elements that may be bundled horizontally and vertically. Excessive bundling may lead to insufficient bond capacity in pretensioned members resulting in bond slip.

CHAPTER 3 - MATERIALS

SECTION 301 - INTRODUCTION

The nature and economics of prestressed concrete construction require the use of high strength materials. Ability to sustain high stresses with a minimum of time-dependent change in stress or strain is essential.

These requirements are more severe than those for conventionally reinforced concrete. Highest standards of manufacture and construction should be observed. Prior to adoption of new materials, sufficient test data should be obtained to verify properties assumed in design.

SECTION 302 - CONCRETE

302.1 SCOPE

Particular attention should be given to properties of individual materials used in prestressed concrete and their effect on compressive strength, modulus of elasticity, drying shrinkage, creep, bond strength and uniformity of concrete in place.

When new materials and methods are employed, trial mix investigations should include tests for drying shrinkage, creep, and modulus of elasticity.

302.2 MATERIALS

302.2.1 Portland Cement

Portland Cement should conform to one of the following:

Specifications for Portland Cement, (ASTM C150)
Specifications for Air-Entraining Portland Cement (ASTM C175)
Specifications for Portland Blast Furnace Slag Cement (ASTM C205)
Specifications for Portland-Pozzolan Cement (ASTM C340)

302.2.2 Concrete Aggregates

Concrete aggregates should conform to one of the following:

Specifications for Concrete Aggregates (ASTM C33)
Specifications for Lightweight Aggregates for Structural Concrete (ASTM C330)

Mineral composition and soundness of aggregates may have a marked influence on compressive strength, modulus of elasticity, drying shrinkage, and creep.

Concretes made with some lightweight aggregates may exhibit a lower modulus of elasticity, greater creep and drying shrinkage than do concretes of the same strength made with aggregates of normal weight.

The range of properties possible in the same concrete mix with different lightweight aggregates may be large. Therefore, it is

recommended that test data should be obtained for compressive strength, modulus of elasticity, drying shrinkage, creep, modulus of rupture and bond.

302.2.3 Water

Water for mixing concrete should be clean and free of injurious quantities of substances harmful to concrete or to prestressing steel. Seawater should not be used for making prestressed concrete.

302.2.4 Admixtures

Certain admixtures may be beneficial to fresh or hardened concrete. However, admixtures should not be used until shown by test to have no harmful effect on the steel or concrete.

The use of calcium chloride or an admixture containing calcium chloride is not recommended where it may come in contact with prestressing steel.

302.3 PROPORTIONING, BATCHING AND MIXING

The proportioning of materials, batching and mixing of concrete for prestressing should be done in accordance with the ACI Manual of Concrete Inspection, the U.S. Bureau of Reclamation Concrete Manual, or other comparable regulations including ACI Standards for Winter Concreting Methods (ACI 604), Selecting Proportions for Concrete (ACI 613), Measuring, Mixing and Placing Concrete (ACI 614), and Standard Specifications for Ready-Mixed Concrete (ASTM C94).

Available materials should be proportioned to produce concrete meeting specification requirements with a minimum water content. Slump of fresh concrete should be as low as feasible. Cement, sand, and narrow-size ranges of coarse, aggregate should be separately batched by weight. Water and some liquid admixtures may be batched by volume with accurate measuring equipment. Close control of all materials and operations is essential.

302.4 STRENGTH

The strength required at given ages should be specified by the designer. Controlled concrete should be used and tested in accordance with Section 304 as modified by Section A602(f) of "Building Code Requirements for Reinforced Concrete" (ACI 318-56)

SECTION 303 - GROUT

303.1 GENERAL

When required by job specifications, posttensioned steel should be grouted to completely fill the void surrounding the prestressing steel with a portland cement grout to insure high flexural bond strength and provide permanent protection for the steel.

303.2 MATERIALS

Grout should be made of either (a) cement and water or (b) cement, fine sand, and water. Mix (a) should be used where the cavity is very small. Either mix (a) or mix (b) may be used where the cavity is relatively large. Admixtures should conform to recommendations of Section 303.2.4.

303.2.1 Portland Cement

Same as Section 302.2.1.

303.2.2 Sand

Sand should preferably be a natural quartz sand meeting Tentative Specification for Aggregate for Masonry Mortar, (ASTM C144), except for gradation requirements. The sand should pass a No. 30 sieve, about 50 per cent should pass a No. 50 sieve, and about 20 per cent should pass a No. 100 sieve.

303.2.3 Water

Same as section 302.2.3.

303.2.4 Admixtures

Certain admixtures may be beneficial to fresh or hardened grout. However, no admixture should be used until shown by test to have no harmful effect on the steel or grout.

Calcium chloride or an admixture containing calcium chloride is not recommended for use in grouting posttensioned members.

303.3 PROPORTIONING

Proportions of grouting materials should be based on results of tests made on fresh and hardened grout prior to beginning work. Grout should have the consistency of thick cream or heavy paint. When permitted to stand until setting takes place, grout should neither bleed nor segregate.

SECTION 304 - PRESTRESSING STEEL

304.1 GENERAL

High tensile strength steel is required in prestressed concrete to provide necessary internal concrete stresses after losses have occurred. The following four types are in common use:

(a) High tensile strength single wire, applied in the form of assemblies made up of two or more substantially parallel wires. They may be used for either pretensioning or post-tensioning purposes.

(b) Small diameter high strength strand, shop fabricated, is usually made up of six wires spiraled around a center wire. Small diameter strand is normally, though not exclusively, used for pretensioning purposes.

(c) Large diameter high strength strand is usually shop fabricated with factory attached end fittings for posttensioned construction. It has 7, 19, 37 or more individual wires.

(d) High strength alloy steel bars are produced by a cold stretching or drawing process. They are currently available in diameters ranging from 1/2 to 1-1/8 in. Alloy steel bars are used principally for posttensioned construction.

Each type of prestressing steel should be made to distinctly separate specifications, of which the following sections give a general description.*

304.2 HIGH TENSILE STRENGTH SINGLE WIRE

High tensile strength single wire is generally made from high carbon steel hot rolled into rods. It is then heat treated by a process termed "patenting" and cold drawn to produce the required final tensile strength. In its most commonly used form the wire is then stress relieved by a controlled time-temperature treatment that improves elastic properties within the tensile range usually employed in prestressing concrete. It also produces a straighter, more easily handled wire.

High tensile strength wire produced by the oil tempering process is not recommended for use in prestressed concrete.

304.2.1 Ultimate Tensile Strength

High tensile strength wire for prestressed concrete is made to minimum tensile strengths as high as 250,000 psi for a diameter of 0.196 in. Higher tensile strengths are available at smaller diameters and lower tensile strengths at larger diameters.

304.2.2 Shape of Stress-Strain Curve

Stress relieved wire for prestressing should display a high yield strength and a reasonable elongation before rupture. Minimum yield strength at 1 per cent elongation under test load should be equal to 85 per cent of specified ultimate tensile strength. Minimum elongation after rupture should be 4 per cent in 10 in. Elongation tests should conform to Specification for Mechanical Testing of Steel Products, (ASTM A370-54T)

304.2.3 Ductility

Wire for prestressing should be capable of a reasonable amount of cold deformation without failure. It should have a minimum reduction in cross-sectional area of 30 per cent at rupture.

304.2.4 Creep and Relaxation

Data concerning typical creep and stress relaxation properties

* The American Society for Testing Materials is currently formulating specifications for prestressing steels.

of the material should be obtained from the manufacturer. Special acceptance tests for individual lots are usually expensive and unnecessary.

Creep tests and short-term relaxation tests do not necessarily represent long-time stress relaxation characteristics.

304.3 SMALL DIAMETER HIGH STRENGTH WIRE STRAND

Small diameter high strength strand is normally made of seven wires. A straight center wire is enclosed tightly by six spirally wound outer wires. Because of its small diameter, strand can be given a final stress-relieving treatment similar to that for single wires. This treatment improves elasticity and handling characteristics. Acceptance tests, when required, should be made on the strand rather than single wires.

Physical properties should be based on the total metallic area of all the individual wires. Ultimate tensile strength, shape of stress-strain curve, ductility, creep and relaxation should be the same as described in section 304.2 "High Tensile Strength Single Wire" except as follows:

(a) Minimum elongation at rupture - 3.5 per cent in 24 in.

(b) Minimum yield strength at 1 per cent elongation under test load equal to 85 per cent of specified ultimate tensile strength.

304.4 LARGE DIAMETER HIGH STRENGTH WIRE STRAND

Large strand may be made of 7, 19, 37 or more galvanized or uncoated hard-drawn wires, spirally wound. Galvanized strand is most commonly used.

Because large diameter strand cannot be given a final stress-relieving treatment, some of its physical properties differ from those of wire or small strand. Acceptance tests, when required, should be based on properties of the strand rather than individual wires.

304.5 COLD STRETCHED HIGH STRENGTH ALLOY STEEL BARS

These bars are usually made from alloy steel designated AISI 5160 or AISI 9260. After hot rolling, the bars are either heat treated or cold worked. Each bar is then cold stretched to a minimum of 90 per cent of the specified ultimate strength.

304.5.1 Ultimate Tensile Strength

High strength alloy steel bars are produced with a minimum tensile strength of 145,000 psi for all diameters.

304.5.2 Shape of Stress-Strain Curve

High strength bars for prestressing should have a minimum yield strength at 0.2 per cent permanent strain equal to 90 per cent of the specified ultimate tensile strength. Minimum elongation after rupture should be 4 per cent in a length of 20 diameters.

304.5.3 Ductility

Bars for prestressing should be capable of a reasonable amount of cold deformation without failure. The bar should have a reduction of area of not less than 15 per cent at rupture.

304.5.4 Creep and Relaxation

Data concerning typical creep and stress relaxation properties of the material should be obtained from the manufacturer. Special acceptance tests for individual lots are usually expensive and unnecessary.

Creep tests and short-term relaxation tests do not necessarily represent long-time stress relaxation characteristics.

304.6 CORROSION

Since prestressing steels are susceptible to corrosion, they should be protected during storage, transit and construction.

The term stress corrosion is applied to the embrittlement of steel that occurs under the combined effects of high stress and some corrosive environments. It may take place without apparent surface impairment.

Normally, steel cast in concrete or properly grouted will not be subject to such corrosion. When posttensioned steel is not grouted, special precautions should be taken to protect the steel, see Section 404.3.2.

SECTION 305 - ANCHORAGES AND SPLICES

305.1 GENERAL

Anchorages for posttensioning elements now in general use consist of:

Threaded ends and wedge anchors for bars; factory attached end fittings for large diameter strand; button-head, sandwich plate and conical wedges for parallel lay wire system; and conical wedges for small diameter strand.

Splices are used primarily for bars and consist of threaded couplings.

305.2 ULTIMATE STRENGTH

Anchorages and splices should be capable of developing the ultimate strength of attached steel elements without excessive deformation.

305.3 ANCHORAGE SET

Movement of prestressing steel in anchorage during seating should be stated by the manufacturer and substantiated by test data.

CHAPTER 4 - CONSTRUCTION

SECTION 401 - INTRODUCTION

This chapter outlines construction procedures that should result in sound and durable structures.

Prestressed concrete members are composed of high strength concrete and steel. Design stresses are closely controlled, but behavior in service depends upon the specified concrete being properly placed in forms of the correct dimensions around accurately positioned prestressing steel or ductwork for steel. Construction requires accuracy and care. Deviation from careful workmanship may result in an unsafe structure and should not be condoned.

SECTION 402 - TRANSPORTING, PLACING, AND CURING OF CONCRETE

402.1 GENERAL

Quality of the finished concrete members depends on care used in transporting, placing and curing. Recommended practice is outlined in ACI 318 "Building Code Requirements for Reinforced Concrete" Sections 403-406, and ACI 614 "Recommended Practice for Measuring, Mixing, and Placing Concrete."

402.2 PLACING

Low slump, high cement content mixes should be placed in the shortest possible time after mixing is completed to prevent loss of workability.

Concrete should be deposited close to its final position. The method of placement should be such that segregation will not occur.

402.3 VIBRATION

Internal or external vibration or both are usually necessary to produce dense, well-compacted concrete.

Vibrators should not be used to move concrete horizontally in the form. Overvibration should also be avoided.

When internal vibration is used, vibrator heads should be smaller than the minimum distance between ducts or prestressing steel. Care must be exercised to avoid damage to or misalignment of ducts for posttensioning steel.

Vibration is not a substitute for workability. Judgment should be used in specifying slump and approved methods of vibration used to achieve maximum compaction.

402.4 CONSTRUCTION JOINTS

In long cast-in-place members the use of construction joints is recommended (1) to reduce cracking near columns caused by settlement or movement of shoring and falsework, and (2) to allow for shrinkage. In general, joints should be placed near falsework supports.

Construction joints preferably should be perpendicular to prestressing steel. Joints should not be made parallel to prestressing steel unless the provisions of Section 212, Composite Construction are followed.

402.5 CURING

Curing should start soon after finishing. If high temperature curing is used, an initial setting time prior to application of heat should be required. Curing should continue until the required strength for application of the prestress force is reached. Fresh concrete should be protected from rain or the rapid loss of moisture prior to the curing period. Rapid drying should be prevented until the final design strength is obtained.

When high temperature curing is used, the rate of heating and cooling should be controlled to reduce thermal shock to the concrete.

Where identical precast members are required, curing conditions should be uniform to maintain proper quality control.

402.6 PROTECTION FROM FREEZING

During periods of freezing temperatures, ungrouted ducts should be blown clear of water or protected against freezing.

SECTION 403 - FORMS, SHORING, AND FALSEWORK

403.1 GENERAL

Quality of concrete members depends on the care used in constructing forms and falsework. Correct practices outlined in ACI 318 "Building Code Requirements for Reinforced Concrete" Sections 501 and 502 are recommended.

403.2 SPECIAL REQUIREMENTS

Forms for pretensioned members should be constructed to permit movement of the member without damage during release of the prestressing force.

Forms for posttensioned members should be constructed to minimize restraint to elastic shortening during prestressing and shrinkage. Deflection of members due to the prestressing force and deformation of falsework should be considered in design. Form supports may be removed when sufficient prestressing has been applied to carry dead load, formwork carried by the member and anticipated construction loads.

SECTION 404 - PLACEMENT OF PRESTRESSING STEEL AND APPLICATION OF PRESTRESSING FORCE

404.1 GENERAL

The location of the center of gravity of the prestressing steel, initial and final prestressing force, and the assumed losses due to creep, shrinkage, elastic shortening and friction shown on the plans

are based on the use of specified materials. Other materials not specified but capable of producing the same results may be used with approval of the engineer.

Unless tolerances for location of the prestressing steel are shown, a variation of $\pm$ 1/8 in. to $\pm$ 1/4 in. depending on size of the member, is suggested as maximum permissible.

404.2 PRETENSIONING STEEL

404.2.1 General

Steel should be kept clean and dry. Foreign matter, grease, oil, paint and loose rust should be removed prior to casting concrete. A light coat of rust is permissible and sometimes preferable provided loose rust has been removed and the surface of the steel is not pitted.

404.2.2 Measurement of Prestressing Force

Pretensioning force should be determined by measuring elongation and checking jack pressure on a calibrated gauge. Measurement of elongation will usually give more consistent results. When there is a difference of over 5 per cent between the steel stress determined from elongation and from the gauge reading, the cause of the discrepancy should be ascertained and corrected.

If several wires or strands are stretched simultaneously, provision must be made to induce the same initial stress in each.

404.2.3 Transfer of Prestressing Force

The force in the prestressing steel should be transferred to the concrete smoothly and gradually. If the force in the wires or strands is transferred individually, a sequence of release should be established by the engineer to avoid subjecting the member to unanticipated stresses. Any variation in this sequence should be submitted to the engineer for approval.

404.2.4 Protection

Ends of pretensioning steel exposed to weather or corrosive atmosphere should be protected by a coating of asphaltic material. They should preferably be recessed in the member, coated with asphaltic material and covered with mortar.

404.3 POSTTENSIONING STEEL

404.3.1 General

The steel should be kept clean and dry. For bonded construction, foreign matter, grease, oil, paint and loose rust should be removed prior to placing steel in ducts. A light coat of rust is permissible provided loose rust has been removed and the surface of the steel is not pitted.

404.3.2 Protection

For general use in unbonded construction galvanizing may be considered to protect the steel from corrosion when coated with grease or asphalt-impregnated material and enclosed in a sheath. Uncoated galvanized steel may be used when it is accessible for inspection and points of bearing are equipped with special shoes to prevent damage to the galvanizing.

If wrappings and coatings are used on non-galvanized steel, the coating should protect the steel from corrosion during shipment, storage, construction, and after the steel is in place. It should permit movement of steel during stressing with minimum friction. The method of protection should be specified or approved by the engineer.

Anchorage and end fittings should be given protective treatment consistent with that given the prestressing steel. They should preferably be recessed in the member and covered with mortar.

404.3.3 Placement of Steel and Enclosures

Ducts or enclosures for prestressing steel are formed in the concrete using tubing, metallic casings or other materials. They should be positioned and secured to maintain the prestressing steel within the allowable placement tolerances.

For bonded construction, ducts or duct-forming devices should be free from grease, paint, or other foreign matter. Ducts should be protected against entrance of foreign matter prior to grouting.

Anchorage hardware to be cast in the member should be firmly fastened to forms in the proper location.

404.3.4 Measurement of the Prestressing Force

Values of total elongation, corrected for assumed friction loss and anchorage set, and corresponding jack pressures at various increments of prestress should be supplied by the engineer. When a difference of over 5 per cent exists between steel stress determined from the corrected elongation and from corresponding gauge reading, stressing operation should cease. If the cause of the discrepancy is neither faulty measurement nor equipment the engineer should be consulted.

404.3.4.1 Factors Influencing Friction

As prestressing force is applied, friction between prestressing steel and curved enclosure reduces steel stress at points away from the jack. The amount of friction loss is a function of degree of curvature, type and length of prestressing steel, duct material, presence of friction reducing agents, accuracy of placing the duct, and degree of disturbance during concrete placement.

It is the responsibility of the contractor to be aware of these factors. He should use materials specified and insure that the quality of workmanship results in accurate duct positioning with minimum displacement during construction.

404.3.5 Prestressing in Stages

When the prestressing force is to be applied in more than one stage, excessive concrete stresses should be avoided during intermediate stages. The engineer will designate location and magnitude of the forces to be used for each stage and allowable external loads that may be placed on the member. The contractor should be aware of the significance of overloading the member.

404.3.6 Anchorage Set

For friction type anchorages the manufacturer or supplier should state the amount of slip normally expected in seating the anchorage device.

404.3.7 Effect of Temperature

Changes in temperature should have little effect on prestressing reinforcement unless there is a significant temperature differential between concrete and steel.

SECTION 405 - GROUTING

405.1 GENERAL

When grouting is specified for posttensioned members it should completely fill all enclosure voids.

405.2 MIXING

Grout should be mixed in a mechanical mixer. Immediately after mixing, it should be passed through a strainer into pumping equipment which provides for recirculation. Grout should be pumped into the duct as soon as possible after mixing but may be pumped as long as it retains the proper consistency.

405.3 ARRANGEMENT OF GROUT PIPES

Ducts must be provided with entrance and discharge ports, each of which can be closed. Extension pipes may be used when necessary.

For long members, grout may be introduced at one end until it discharges from an intermediate point. The point of application may then be moved successively forward. Grout may be introduced at an intermediate point if discharge ports are provided at duct ends. The sequence of grouting should be planned to insure complete filling. Devices for bleeding air may be required at high points of the duct profile.

405.4 TEST FOR PASSAGE OF GROUT

Free passage of grout from entrance to discharge port must be assured. Tests may be made by pumping water, air or other fluids through the duct.

405.5 APPLICATION OF GROUT

Grout should be applied continuously until it flows steadily from the discharge port indicating removal of trapped air and water. The discharge port should then be closed and grouting pressure maintained for the length of time necessary to insure complete filling of the void. The entrance should then be closed and the pumping nozzle removed.

405.6 PROTECTION AGAINST FREEZING

Adequate precautions must be taken to prevent freezing fresh grout.

SECTION 406 - HANDLING AND ERECTION

Where precast members are specified, methods of handling and/or the sequence of erection should be indicated. When these are not indicated on the plans, the contractor should submit for approval the location of pick-up points, minimum concrete strength when handled, method of transporting, and sequence of erection.

Respectfully submitted,

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PRESTRESSED CONTINUOUS BEAMS AND FRAMES[a]

Closure by P. B. Morice and H. E. Lewis

P. B. MORICE[1] and H. E. LEWIS.[2]—It is interesting to have the results of Dean Kalmas' calculations for a non-uniform loading on three span continuous beams. Certainly the writers' study was restricted to a uniformly distributed loading applied to a beam of uniform cross-section and was designed to explode what had become a commonly accepted fallacy.

The tests of Dr. Mattock and Mr. Kemp on the ultimate strength of prestressed portals do not strictly fall within the scope of the paper but they are nevertheless important in the problem of the design of prestressed concrete structures. Whilst the writers are not entirely in agreement with them in their description of the approach to the collapse mechanism, both their results and our results, to which they referred, do adequately support the theory that a very large degree of moment redistribution occurs in prestressed concrete and that linear transformations of tendon profiles do not have any significant effect upon the ultimate strength of such structures.

As the writers have reported,[3] the rotation in a section with a high steel percentage, resulting from transformation towards the compressive face (i.e. reduced effective depth), does not depend solely upon reaching high compressive concrete strains. The important feature is that the effective depth is much reduced. In fact bigger rotations occur in an over-reinforced section, which is over-reinforced because the steel is placed near the compression face, than in the same gross section when the same quantity of steel is placed near the tension face. The discussers (Fig. 3) shows cracks which have every appearance of being tensile cracks. It is only when one remembers the position of the steel that it is realized that they are associated with a compression failure.

---

a. Proc. Paper 1055, September, 1956, by P. B. Morice and H. E. Lewis.
1. Cement & Concrete Assn., London, England.
2. Cement & Concrete Assn., London, England.
3. Morice, P. B. and Lewis, H. E., "The Strength of Prestressed Concrete Continuous Beams and Simple Plane Frames." Symposium on the Strength of Concrete Structures. Session C. Paper No. 3. London 1956.

EFFECTS OF CAMBERING OF STEEL WF BEAMS[a]

Closure by Harry H. Hill

HARRY H. HILL.[1]—Mr. Edwin W. Thomas' comments and suggestions are heartily welcomed. He has pointed out an error in notation which should be corrected. The simplest manner to accomplish this would be to change the "y" and "dy" in the left hand portion of Fig. 1 to a "z" and a "dz". This would then define the distance from the neutral axis to any fibre as "z". The only equations to be changed by this would be Equations (3) and (4) and are corrected as follows:

$$M = 2\int Szdz \tag{3}$$

$$M = 2\int_{0}^{c-h} \frac{y}{c-h} S_e zbdz + 2\int_{c-h}^{c-t} S_e zbdz + 2\int_{c-t}^{c} S_e zb'dz \tag{4}$$

The inclusion of the application of the equations immediately following the derivations was for the purpose of explaining the terms in the final form of the equation. However, should there be any question remaining, Equation (5) is a general expression for the resisting moment due to inelastic action. Equation (8) is a general equation applicable to any WF beam and Equation (24) is the general equation for deflection due to the assumed loading condition.

The writer, at the time of preparing the paper, was aware of the procedure of computing deflections as suggested by Mr. Thomas. This method was seriously considered for use and although the method presented in the paper is cumbersome, it was felt that it was more easily adapted to the problem at hand.

In conclusion, the writer wishes to thank Mr. Thomas for his interest in the problem and for his aid in clarifying definitions and explanations for other readers.

a. Proc. Paper 1146, January, 1957, by Harry H. Hill.

1. Asst. Prof. of Mechanics, University of Oklahoma, Norman, Okla.

A CIRCUIT ANALYSIS OF LATERALLY LOADED CONTINUOUS FRAMES[a]

Closure by Frank Baron

FRANK BARON.[1]—The writer thanks Mr. Nubar for his discussion in regard to the analytical procedures presented in the paper.

At the outset it should be mentioned that Mr. Nubar's discussion is concerned chiefly with the two sections of the paper that deal with a distribution procedure for laterally loaded continuous frames. Of equal interest is the preceding section wherein the column analogy and the shear and torsion analogy are extended to structures having curved or segmental members lying in a plane and forming any number of multi-connected circuits in the plane. An inspection of this section shows that for each class of loading a solution is obtained in two parts. The first part consists of any statically possible distribution of shears and moments consistent with the loads on the structure. The second part consists of a pattern of corrections that is self-balancing and is defined by expressions similar to those of the column and the shear and torsion analogies for single closed circuits. These expressions are given in Equations 11a to 11c of the paper. Particular attention is called to the distribution of correction shears and moments that is defined in the members of a closed circuit by each of these expressions. The distribution in each case is planar as can be seen by inspection of Equations 4a to 4c. This is the same as in the respective analogies for single closed circuits.

The writer believes that the extensions of the two analogies to multiply-connected circuits are important. The extensions are generalized for structures lying in a plane and can serve as bases for devising any number of ways to satisfy the requirements of geometry for each circuit. It is emphasized that the requirements of geometry are stated for closed circuits and not for linear paths or at junctions of members. Consequently, various procedures can be developed that are based on considerations of the flexibilities of panels and not of the stiffnesses of members meeting at junctions. The distribution procedure presented in the paper is one of these ways.

As to Mr. Nubar's comments in regard to convergence of the distribution procedures, the writer agrees that no general proof or criteria of convergence exists for processes of this type. The proofs that exist are restricted to special groups of problems. The writer concerned himself with convergence in his "General Discussion" and in the selection of the numerical examples. Mr. Nubar will recognize that the paper presents a generalized procedure which is illustrated by specific examples. Here, as elsewhere, the matter of convergence is best pursued for specific families of problems or groups of structural configurations having some things in common. The

a. Proc. Paper 1147, January, 1957, by Frank Baron.

1. Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

writer suggests that this matter be pursued for various classes of structural types. The question of convergence will then be answered in different ways depending on (1) preferences in selecting the initial distributions of shears and moments consistent with the requirements of statics, and (2) preferences in selecting a sequence for satisfying the requirements of geometry for each closed circuit.

As to speeding up convergence for those cases wherein the influence of load concentrations is desired to be diffused rapidly to adjacent areas, the same comments as above apply. This same question has been raised in regard to other distribution procedures. The answers and procedures developed therein can serve as guides to the present procedure.

Concerning the condensed form of the paper, the writer accepts Mr. Nubar's comments as a compliment. In turn, he sincerely compliments Mr. Nubar in demonstrating that the paper although brief is complete. Mr. Nubar has demonstrated that the necessary details can be filled in by the reader. As to his question concerning frames with curved members, the procedure takes into account the change in length between the ends of members in the same way as any other procedure that is based on the concept of small deformations. In this theory, the computations for forces and moments in a structure are based on the initial dimensions of the structure. It should be recognized that the present procedure is not based on the assumptions of a deflection theory.

Mr. Nubar's comments pertaining to programming procedures for use of high-speed computers are appreciated. These comments are particularly pertinent at a time when much interest is being expressed in programming operations. Mr. Nubar indicates that preferences exist herein as elsewhere and need to be examined before suitable programs can be defined. The writer agrees with these views. In addition, he agrees that for use of high speed computers a method of solution by matrix inversion may be more advantageous than by iterations. In that case, the required equations of geometry are already written. These are Equations 13a to 14c of the paper. The writer wishes to add that much of the work needed for programming the procedure for a specific case is also performed. This is performed in the summaries and tabular forms of Figs. 5, 6, and 7. It remains to translate these into an instruction code for a machine.

Mr. Nubar describes another method that can be used for the analysis of frames loaded in their own plane. The method is of interest. However, it will not be commented upon as it is not pertinent to the present discussion.

In conclusion, the writer wishes to emphasize an important attribute of the procedure as given in the paper. Although the procedure has been described formally, it need not be performed that way. The procedure can be performed informally in obtaining an estimate of structural behavior. For a given structure, guess at a distribution of shears and moments that satisfies the requirements of statics. (This distribution can be guided by visualizing, or sketching, the distortions of the deflected structure. It is observed that the distributions of moments are proportional to the distortions of the structure and the stiffnesses of the elements.) Then check the requirements of geometry and revise the estimate if the estimate is considered unsatisfactory.

Thanks again are given to Mr. Nubar for his discussion. His comments were constructive and helpful.

SHEARING STRENGTH OF REINFORCED CONCRETE SLABS[a]

Closure by Nan-Sze Sih

NAN-SZE SIH,[1] A.M. ASCE.—The writer wishes to express his appreciation for the careful and critical review of his paper.

The factor C_b is a non-dimensional coefficient, the numerical values assumed in the paper is only for qualitative comparison of the test results. However, in order to be consistent with the results of Fig. 2, $C_b = 2.4$ may be assumed for the rectangular slab tests, rather than the value of 3 as pointed out by the discussers. With $C_b = 2.4$ the results of the rectangular slab tests still points to the existence of such a factor.

On page 1149-2 it was stated that the factor $(1 + C_b\frac{d}{b})\,b$ represents the area of the base of the concrete cone that was pushed out from the slab. Perhaps the word "area" should be changed to say "size". Since $(1 + c_b\frac{d}{b})\,b$ is a linear factor. This may eliminate several misunderstandings pointed out by the discussers.

There are two errors in the paper the author wishes to make a correction. Firstly, in Fig. 3, where a symbol which is circled means $f_s < f_{yp}$ not $f_s = f_{yp}$. Secondly, on page 1149-8 at the right hand side of Equation (4) there is an m_o omitted from the denominator. However, there is no m_o factor on the right hand side of Equation (5), since in this case $\frac{P_{ult.}}{bd(1 + C_b\,d/b)}$ is independent of m_o.

In conclusion, the purpose of the paper is not merely to establish two emperical equations as a result of the study of the various uncoordinated tests. It is to point out that clear understanding of the basic characteristics of concrete and the problem involved, are more important toward the solution of the problem.

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a. Proc. Paper 1149, January, 1957, by Nan-Sze Sih.

1. Asst. Structural Engr., Ammann & Whitney, Cons. Engrs., New York, N. Y.

BEHAVIOR OF RIVETED CONNECTIONS IN TRUSS-TYPE MEMBERS[a]

Closure by E. Chesson, Jr. and W. H. Munse

E. CHESSON, JR.[1] J.M. ASCE and W. H. MUNSE,[2] A.M. ASCE.—The writers would like to thank Professor A. J. Francis for his discussion of their paper and the thoughts he has presented concerning the behavior and design of riveted joints. It is believed that this discussion adds much to the value of the paper, particularly the comments concerning the deformations in joints with punched holes.

The more thorough theoretical explanation of the behavior of the truss-type joints suggested by Professor Francis will be a part of the next phase of the research program in which the large connections are being tested. Twenty-eight additional large riveted connections are now being tested to answer many of the questions raised as a result of the initial tests. It is hoped that upon completion of these tests, sufficient data will be available to permit a more sound evaluation of the applicability of previous theoretical analyses or the development of an analysis which will explain effectively and accurately the behavior of large riveted connections. However, since most of the previous theoretical studies have been based upon an elastic analysis, it is doubtful that they will provide the desired correlation with the results of the laboratory tests conducted to failure.

Several of the findings reported by the writers and discussed briefly by Professor Francis are being studied further in the current program of tests. These studies include, (1) tests to confirm the importance of distributing the fasteners on the basis of the distribution of the cross-sectional area, (2) further tests of "punching vs. drilling", and (3) tests to evaluate the behavior of truss-type joints assembled with high strength bolts: some with the shear area smaller than that now required by the current specifications. The data from these various tests, when combined with the other data currently available, should help greatly in providing the information necessary for an effective re-evaluation of our design requirements for riveted and bolted connections which are subjected to static-type loadings.

---

a. Proc. Paper 1150, January, 1957, by E. Chessen, Jr. and W. H. Munse.

1. Research Associate, Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.
2. Research Prof., Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

CHOICE OF COMPOSITE BEAMS FOR HIGHWAY BRIDGES[a]

Closure by Harry Subkowsky

HARRY SUBKOWSKY,[1] A.M. ASCE.—By rearrangement of the data presented, Mr. Zalite has presented a chart which more readily permits the selection of various beam and cover plate combinations for composite, simple span, highway structures. The elimination of thickness of cover plate as a factor in the determination of the required area is theoretically incorrect. For practical problems, however, the areas of cover plate selected from Mr. Zalite's chart are sufficiently accurate.

By eliminating the use of the "K" factor, Mr. Zalite's chart permits the selection of beam and area of cover plate only. Once these are selected, the designer must then determine the properties of the composite section to find the length of the cover plate and of the spacing of the shear connectors.

By retaining the "K" factor, it is possible to use Figs. 5, 6 and 7 to select the length of cover plate and shear connector pitch directly.

The values of the length of cover plate determined from Fig. 5 must of course be corrected to reflect the design tensile stress being used in the beam at the end of cover plate. The value to be used for the permissible stress may however vary from the 13,500 psi used by Mr. Zalite. The design practice of the New York State Department of Public Works for example, is to use 16,000 psi as the limit for this stress.

With reference to Fig. 7, the static moments Q shown apply to the composite section with beam and cover plate. The value of Q for the slab and beam without cover plate is a constant for each beam selected with the 84" x 7" slab and is applied in determining the pitch at the section where there is no cover plate. On the portion of the beam where a cover plate is used, the value of Q as shown in Fig. 7 and I from Fig. 6 is used to determine the shear connector pitch.

Mr. Zalite's Example A is correct in pointing out that a 36 WF 150 beam with a 18.8 square inch cover plate will satisfy the span and spacing requirements more economically. The same value may be determined by use of Fig. 2 if the curve for the 7.5 foot spacing is sketched in and applied instead of using the projected curve for the heavier beam section and interpolating for the value of "K".

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a. Proc. Paper 1151, January, 1957, by Harry Subkowsky.

1. Supervising Structural Engr., Goodkind & O'Dea, New York, N. Y.

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CONFIRMATION OF INELASTIC STRESS DISTRIBUTION IN CONCRETE[a]

Closure by Eivind Hognestad

EIVIND HOGNESTAD,[1] A.M. ASCE.—Mr. DiGioia suggests that a rectangular distribution of concrete stress in ultimate strength design is not logical and is incompatible with a linear strain distribution. He fears, therefore, that use of the rectangular distribution in design involves a risk that reality may be lost to such an extent that inadequate routine procedures become used.

It is important to note in this connection that the rectangular stress block is only an equivalent stress distribution. In most cases of practical design, it leads to sufficiently accurate evaluations of ultimate moments and loads. For non-rectangular cross-sections, the rectangular stress block is considerably easier to use than trapezoidal or parabolic stress blocks. However, the approximate nature of the rectangular block should always be kept clearly in mind. For unusual cases, such as a cross-shaped section with the horizontal bar of the cross located close to the neutral axis, calculations by the rectangular stress block may wisely be checked using a more realistic stress distribution.

a. Proc. Paper 1189, March, 1957, by Eivind Hognestad.
1. Portland Cement Assn., Chicago, Ill.

LATERAL LOAD DISTRIBUTION TEST ON I-BEAM BRIDGE[a]

Discussion by Louis Balog

LOUIS BALOG.[1]—Measurements on actual bridges disclose their behavior as compared to the performance aimed at in their design, therefore, they can help in improving the layout and design of bridges. This paper describes the testing of a three-span, continuous, partially composite, deck bridge having five haunched girders framed by the slab and cross frames. The aim of the authors was to determine the degree of composite action obtained and the effectiveness of the diaphragms in lateral distribution of loads. The results of these tests as they effect design, will be discussed in the following:

Layout

The design specifications of the American Association of State Highway Officials (AASHO) require that concrete on the compression side of the neutral axis only shall be considered in computing moments of inertia and resisting moments. Continuous girders are designed for composite action only in the lengths of their positive moments, as a rule. The magnitude and lengths of the negative moments increase and of the positive moments decrease with increasing depth and length of the haunches of the continuous girder. These properties of such girders as they effect their behavior in composite construction raise interest in comparative economic studies.

Such studies may include the placing of part of the top flange area of the constant-depth girder into the slab in the form of reinforcing bars in the negative moment regions. They may include the provision of a bottom slab in the support region of haunched or constant depth girders, assuring full-length composite action. For shallow spans, solid sections over the supports may also be considered.

It is characteristic for the proportions of the measured girders that due to the loads at midspan, Section A, the measured maximum compressive concrete strain was exactly of the same magnitude as the measured maximum tensile concrete strain at Station B, over the support. For girders of constant depth, the support moment can be about half as large as the midspan moment due to this loading. These data indicate the comparative shortness of the lengths of the composite design regions of the deep haunched continuous girder.

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a. Proc. Paper 1255, May, 1957, by Ardis White and William B. Purnell.

1. Cons. Engr., Binghamton, N. Y.

The Degree of Composite Action Obtained

The highest stress that occurred during these tests was about one-third of the allowed design stress. In this stress range the bond between the concrete and the steel top flange was sufficient to transmit the shear without the aid of the connectors. Measurements on a large number of bridges showed that bond maintained their composite action under traffic and temperature effects for decades.

The more frequent use of composite design began about twenty years ago. A large scale application was at that time the 161,000 sq ft composite floor structure of the Rhine River, Köln-Rodenkirchen, suspension bridge having a 1,240 ft long center span. A full size floor panel, 34.45 ft by 30.01 ft, was extensively tested. These tests revealed incomplete composite action, therefore, supplementary beam tests were made by Prof. Graf in 1938 to determine the most efficient connection between the 11 in. deep cross beams and the 7.87 in. thick concrete slab. Fig. 1 shows the results of these tests based on which Type A connector, a uniformly roughened top flange surface, with the addition of slab anchors, was built.(1)

Type A test beam failed by shearing the concrete without any damage to the bond between the slab and the top flange of the beam. All other types of test beams failed at the plane joining the slab and the top flange of the beam. The top surface of this flange received a coat of bituminous paint in test beams Types C and D to eliminate bond in testing the effects of the connectors only.

The authors found that complete composite action at Station A about doubles the moment of inertia of the 33 WF 141 beams and that, for all practical purposes, no slip between slab and beams occurred even at the points of negative moment. Analyzing the measured deflections and the measured vertical frequency of the Xenia, Ohio, Bridge (Fig. 2), in 1945, W. S. Hindman and L. E. Vandegrift found that composite action raised the moment of inertia of the 36 WF 230 beams 2.41 times, without shear connectors.(2) Bond automatically provides composite action and connectors and anchors assure its permanence. Although composite action and for it advantageous stress conditions can be achieved simply, domestic practice utilizes it only for the "suspended spans" in the continuous girder.

Load Distribution to Beams

The authors' test results reveal distribution diagrams which are nearly straight lines. Unfortunately, the median strip introduces an additional vertical and torsional restraint which eliminates symmetry. The distribution pattern is similar to that created by a perfectly rigid center diaphragm with the addition of a certain torsional resistance. This pattern will change but little if the moment of inertia of the center diaphragm is 0.2 of that of the beams, or larger. Therefore, the finding of the authors that the outside beams carry live load of the same magnitude as the inside beams, results also from the simplest computations. The AASHO specifications, however, assigned 1 P and 1.5 P design live load to the outer and inner beams, respectively, of this bridge.

The authors distributed the moment among the beams according to the measured strains and obtained for the simultaneous application of the three

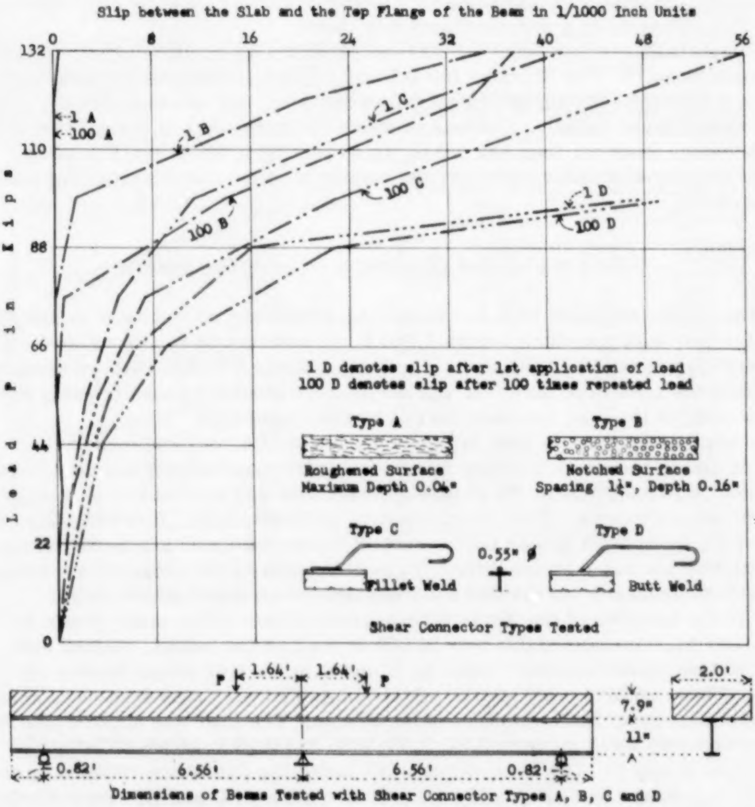


FIG.1 - Rhine River Bridge at Köln-Rodenkirchen - Composite Beam Tests in 1938

loading conditions, 62, 69, 62, 57 and 50 percent of the total moment for beams 1 to 5, respectively. Assuming similarity between the measured deflections and the share of the beams from the loads, 66, 66, 61, 56 and 51 percent load for beams 1 to 5, respectively, result. The agreement is satisfactory, although these percentages may change somewhat in computing the moments. A perfectly rigid center diaphragm results in 60 percent load on all the beams, or in a practically usable value.

Essentially similar behavior was indicated by the measurements on the Xenia Bridge.(2) Fig. 2 shows the influence lines of load distribution of this bridge, as derived from the measured deflections, and as computed for I_c/I_g ratios of 0.2 and infinity. Table 1 contains the evaluation of these three kinds of influence lines for four unit loads, representing H 20-44 truck loading. Multiplications of these values by the proper moment coefficients give similar results.

Effect of Slab and Diaphragm on Load Distribution

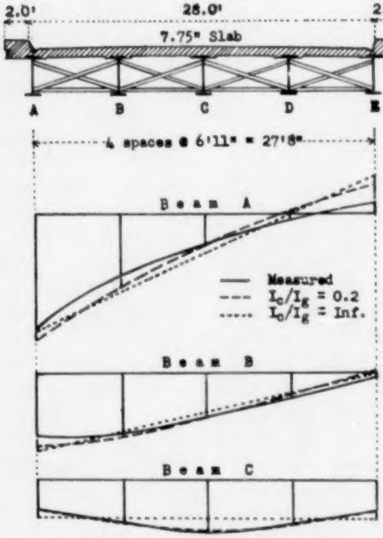
The combined action of the slab and the diaphragm at Station A in distributing the test load placed on beams 1 and 2, as determined by the authors, is in general agreement with the measurements at Xenia.(2) The authors computed from the measured stresses and the properties of the cross framing that 20 percent of the load was transferred by the diaphragm.

In measuring the stresses for three separate identical loadings in the beams and in the cross framing members simultaneously, the authors found considerable variation in the diaphragm stresses and practically no change in the beam stresses. This observation is generally true. If the rigidity of the cross framing is larger than a certain value, the beam stresses remain practically the same irrespective of great changes in the cross frame stresses either because of increasing their number or sectional properties.

The top member of the Xenia Bridge cross frame is the slab. Beam deflections were measured due to a 20,000 lb load on the beams, without and with bottom struts between beams B, C and D in the five cross frames per span. These added twenty struts, which completed the cross frames, contain 0.3 of 1 percent of the total steel in the bridge. The load was applied and the measurements were made at 0.21, 0.40, 0.58 and 0.76 span length points of beams A, B and C. The beam deflections under the load were reduced by the struts in average by 4, 9 and 14 percent for beams A, B and C, respectively. The average reduction in the deflection diagram areas for all beams was 7.5 percent. The efficiency of the completed cross framing in reducing deflections increased toward the center of the roadway and changed little along beams B and C. At the 0.40 and 0.58 span length points of beam A, the deflections under the load were reduced 5 percent, at the 0.21 and at the 0.76 span length points the completed cross frames changed the deflections slightly.

For comparison with the authors' findings, the effect of the cross frames at the 0.40 span length point with a 20,000 lb load on beams A and B will be indicated. Assuming that the deflections of the beams are proportional to the loads they carry, the loads on beams B and C changed by 2,020 lb because of the erection of the struts. This is about five percent of the total load which causes 1,850 psi stress in the cross frame diagonals between beams B and C.

Considering composite action for both the girders and the diaphragm at



Influence Lines of Load Distribution

All Beams 36 WF 230

Five intermediate cross frames per span spaced 15.5 ft. All cross frame members 1 angle 3 x 3 x 5/16, welded connections.

Ordinates of Load Distribution Diagrams

		From Measurement	$I_c/I_g = 0.2$, $z = 35$	I_c/I_g = Inf.
Beam A	A	0.578	0.646	0.600
	B	0.309	0.375	0.400
	C	0.152	0.153	0.200
	D	0.022	-0.016	0.100
	E	-0.061	-0.158	-0.200
Beam B	A	0.322	0.375	0.400
	B	0.309	0.319	0.300
	C	0.220	0.220	0.200
	D	0.127	0.102	0.100
	E	0.022	-0.016	0.000
Beam C	A	0.152	0.153	0.200
	B	0.215	0.220	0.200
	C	0.266	0.254	0.200
	D	0.215	0.220	0.200
	E	0.152	0.153	0.200

$z = (1/2a)^3 \cdot (I_c/I_g)$

FIG. 2 - Xenia Bridge, Ohio, Measured and Computed Diagrams of Load Distribution

TABLE 1 - Xenia Bridge - Evaluation of Load Distribution Diagrams for H 20-44 Truck Loading

	Beam A		Beam B		Beam C	
	$U^{(1)}$	$U^{(\lambda)}$	$U^{(1)}$	$U^{(\lambda)}$	$U^{(1)}$	$U^{(\lambda)}$
Measurement	1.014	-0.303	1.036	0.362	0.921	0.501
$I_c/I_g = 0.2$	1.152	-0.441	1.043	0.287	0.917	0.450
I_c/I_g = Inf.	1.243	-0.533	1.022	0.376	0.800	0.622

Station A, an I_c/I_g value of 0.2 results. Such a ratio for the girders and the combined action of five cross frames of the Xenia Bridge is about 0.1. The combined action of five cross frames was considered by doubling the moment of inertia of one frame. This factor 2, for expressing the effect of the other 4 cross frames in the span, was found by Fritz Leonhardt empirically.(3)

The steel content of the cross frame at Station A, is four times that of a frame at Xenia. Using the slab as top strut for the frames not only saves expense but it also eliminates shrinkage stresses, this arrangement has been frequently used also for larger bridges. That a concrete slab efficiently distributes loads among steel beams was proved by field measurements and computations in 1928, however, the omission of cross frames in bridges never became a general practice.

The German highway bridge and composite highway bridge specifications (DIN 1075, 4.3 and DIN 1078, 7.8, respectively) permit the omission of load distributing cross framing if the stresses in the slab because of its load distributing action are considered in design for the most unfavorable loading condition. The AASHO specifications on the other hand, require that the spacing of substantial cross frames or diaphragms shall not exceed 25 ft. This requirement may be severe and is indefinite. The data in Fig. 2 show that a center diaphragm with an I_c/I_g ratio of 0.2, distributes the loads among the beams of 85.5 ft span without the deck slab in practically the same manner as the slab and the five cross frames. As long as the slab acts as a distributor, however, the stresses in the framing are of the order as measured by the authors. It would seem that the specifications are inconsistent in stipulating constant load distribution and also substantial framing at less than 25 ft.

Full Size Tests and Bridge Measurements

Experience demonstrated that small scale models are insufficient for establishing the true behavior of composite construction. In connection with the design of the Danube River "Arpad" Bridge Budapest, 90.55 ft wide, 4-girder, continuous deck structure, 197 ft to 338 ft spans, 280,000 sq ft floor area, construction of which began in 1938, five kinds of full size, 24.8 ft by 30.5 ft, floor panels were tested. Two of these were arched plate, two were flat plate, composite floors completely welded with stringers spaced 4.6 ft, and a concrete slab composite floor with stringers spaced 5.7 ft.

The ratio of the allowed design stress to the measured maximum stress under design loading in the stringers varied between 1.05 and 2.78. The stresses computed from the measured strains were 7 to 10 percent larger than the stresses computed from the deflections. It was found that the measured maximum stress in the stringers was not only a function of the value of $z = (1/2a)^3 (I_c/I_g)$, but also of the ratio of the number of wheel loads and the number of the stringers. The measured distribution of the loads was better than computed.

Complete composite action was achieved under static and dynamic fatigue testing until the span of the specimens was increased to 29.6 ft and they were set into oscillating motion when the original static deflections increased 7 to 15 times. After several thousand loading cycles in this third phase of the oscillograph tests the specimens began to fail. During the previous two phases of oscillograph testing, 6-million cycles in each, no failures occurred.

Based on the results of these tests the composite concrete floor was selected for the larger part of the structure, and the originally designed

stringers were substituted by 24 percent lighter beams. These tests disclosed the static, dynamic and fatigue behavior of all parts of the structures, including the protective concrete layer and asphalt topping. All these data can be obtained only by full scale testing, which yielded savings and reliable proof about the performance of the floor.[4]

Fig. 3 shows the results of the deflection and stress measurements on December 8 and 9, 1949, on the 88.48 ft suspended span of the Friedrichs Bridge in Heidelberg, Germany, 9 precast girders framed by a full depth center diaphragm and by two thinner half-depth diaphragms at the quarter points. The predetermined accuracy of these measurements was within 6 percent.[5]

Simultaneously with the deflections strains were measured at 13 locations of the specially exposed reinforcing steel at the bottom of the girders and center diaphragm. Two 44,092 lb loads were centered on two girders, except for girders G and H on which the loads were slightly inward. The measurements showed that precasting the girders had no influence on the distribution of the loads, which was better than computed. The outside girders which would be stressless according to the AASHO specifications, had the highest stresses. The stresses of the inner girders under the loads varied little. It was found that the measured stresses correspond to the uncracked condition of the concrete in every case. These measurements were similar to those of the authors in that two girders were loaded simultaneously.

Fig. 4 shows some of the results of the measurements on a 4-span continuous welded composite two-girder bridge in 1938-39. The data in Fig. 4 show that the maximum stress due to 2.5 kpf live load, 127 psf of roadway, was about one-quarter of the allowed stress, furthermore, that the reduction of the stresses is much larger at the top flange than at the bottom flange of this symmetrical girder section. Over the pier the stresses are less than 1/10 of the allowed and the top flange stresses are somewhat smaller than the bottom flange stresses. The deflections are half of the computed values.

SUMMARY

Among the bridges discussed herein only the Xenia Bridge has no slab connectors, nevertheless, its measured composite efficiency was greater than of the others. Indeed, such behavior of many measured bridges lead to the use of slab connectors and to the consideration of composite action in design. However, because of the composite action of bridges without connectors, those with connectors but no other features, differ only in that they are higher stressed and less rigid. By additional features and manipulations a truer structural efficiency can be achieved.

Load distribution is effected by transverse or horizontal framing also by the torsional stiffness of the girders or that of the entire cross section. The framing effect of the slab can be achieved also by concentrated cross frames alone. Top and bottom slabs, or torsion stiff sections, distribute loads without diaphragms. Clarity about the action of all these features leads to efficient design.

Composite action is not isolated to the slab and girders. Transverse and lateral framings effect the deformations, stresses and vibratory characteristics of bridges in various extents. Rational design exploits all features which significantly effect bridge behavior. The measured static and dynamic behavior of bridges is the only scale for the value of the method applied in their

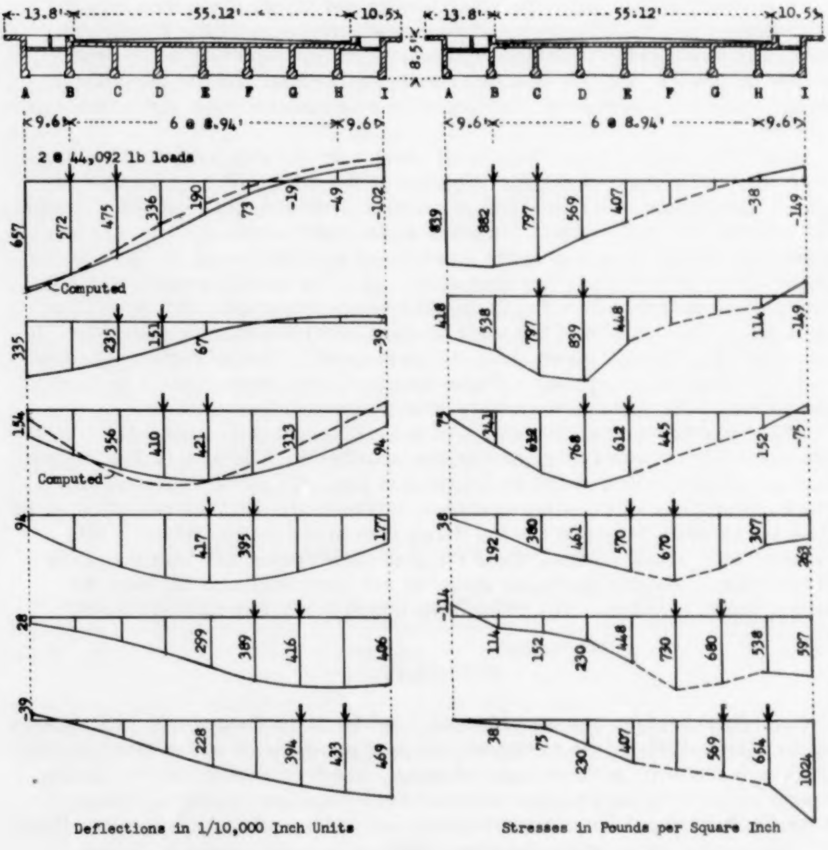
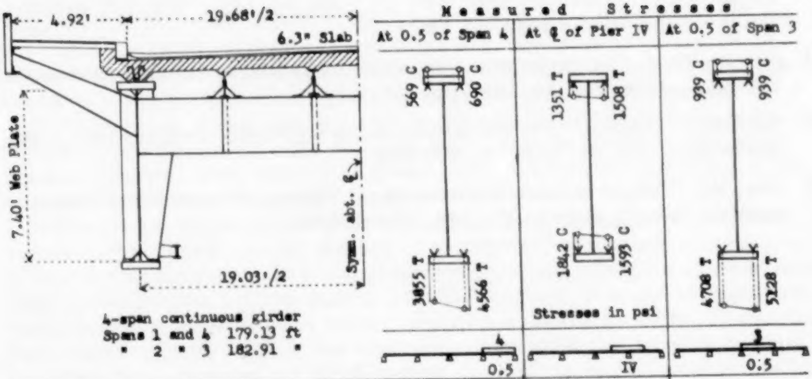


FIG.3 - Friedrichs Bridge, Heidelberg, Germany - Measured Deflections and Stresses



	At 0.5 of Span 4	At ℄ of Pier IV	At 0.5 of Span 3
Reduction of stresses by composite action			
Top flange, Percent	83	48	86
Bottom flange "	21	37	26
Average "	52	43	56
Deflection without composite action, In	2.04	-	2.06
Measured deflection "	1.00	-	1.03
Reduction of deflections Percent	51	-	50

FIG.4 - Sava River Bridge, Zagrab, Yugoslavia - Measured Deflections and Stresses

design. The correlation of measured deflections and vibrations with the quantities governing the design is the most convenient for design purposes.

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VEHICLE LOADS AND HIGHWAY BRIDGE DESIGN[a]

Discussion by Douglas T. Wright

DOUGLAS T. WRIGHT.[1]—In this time of great highway expenditures, when road systems are being built that are presumably intended to give service for many years, the points raised in this paper are of great importance. If, as the authors suggest, current bridge design practice leads to variations in levels of "safety" (however defined) from span to span, engineers 25 years hence will have a difficult job trying to make the best of our bequests.

Unfortunately, in carrying out routine designs of highway bridges according to established procedures, the inconsistencies in these procedures are not readily apparent. Only when studies are made of overload capacities, to determine what bridges can really carry, are the troubles seen.(1, 2)

Although the writer agrees with the authors as to the factors contributing to this serious situation—the manner of specifying loadings and stresses, he is inclined to disagree with some of their suggestions. In spite of the present large investment in bridges built to H20-S16 loadings, it is not reasonable to conclude that a larger design loading would be undesirable except by an argument based on needs. Since our experience does show that modern bridges, including those designed for H-15 loading, can, in one way or another (perhaps with some "over-stressing"), regularly carry modern heavy vehicles we can perhaps conclude that bridges as strong as these are in fact satisfactory. But how strong are these bridges? It is easy to show by any realistic approach that real capacities are much greater than the nominal design loading. Surely then we can approach a more rational design procedure by determining the maximum real strength provided by current methods, and using this as a design loading coupled with rational methods of considering the relative importance of dead and live loads etc. (as discussed by the authors). Such a procedure would result in bridges of roughly the same real strength as the strongest now built—a strength that experience indicates as adequate. More importantly such a procedure would result in far more uniform margins of "safety" that present practices provide. And last but by no means least such a procedure would give more nominal strength with no increase in cost.

The second point of the author's summary, that stress repetitions are not important for the heaviest loads encountered seems to invite an argument for plastic methods of design for bridges. Although plastic methods are not generally suitable for repetitive loadings, if the numbers of maximum loads are small, shakedown occurs and fatigue is insignificant. There is always an economic advantage in plastic design of indeterminate structures and it is not

a. Proc. Paper 1302, July, 1957, by S. Mitchell and G. F. Borrman.

1. Dept. of Civ. Eng., Queen's University at Kingston, Ontario, Canada.

inconceivable that bridges might be designed such that elastic behaviour was preserved for the most severe probable combination of random heavy vehicles whilst plastic behaviour was tolerated for extraordinarily heavy vehicles, those that occur only very infrequently.

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DISTRIBUTION OF LOADS ON BRIDGE DECKS[a]

Discussion by Z. S. Makowski

Z. S. MAKOWSKI.[1]—Mr. Lount's paper touches an extremely important problem of the lateral load distribution in bridge decks. A survey of the bibliography on the stress analysis of interconnected bridge girders indicates that this problem has attracted the attention of many distinguished mathematicians and practical engineers for the last 60 years or so, and still does.

Its importance is now realized fully by many bridge designers, owing to the sudden increases in the magnitude of indivisible loads, carried nowadays by the modern bridges and also owing to the fact that during the last 20 - 25 years the ratio of the useful live load carried by the bridge decks to their dead load has been steadily increasing. The modern bridges are now much lighter, due to the introduction of new structural materials, e.g. aluminium or high tensile steels, or due to new developments in more rational use of old materials such as the prestressing techniques of the reinforced concrete bridges. The considerable reduction in the dead load means that, now, modern bridges are more than ever susceptible to the action of moving loads.

The general shortage of steel and the constant increases in prices of building materials demand design methods which would produce not only safe bridge structures, but also economical ones.

The precise analysis of bridge structures in which the interconnection between the main girders and the transverse diaphragms is taken into account, always leads to a saving of material.

It is interesting to note that although in Europe the problem of the load distribution on bridge decks was discussed in great length by many eminent writers, in the U. S. A., for various reasons, only very few engineers turned their attention to the importance of this problem. One of the very first American engineers who fully realized the importance of the lateral load distribution in the interconnected bridge systems and also the resulting consequences in the design seems to be L. Balog, who, in 1949, emphasized the beneficial effects of the transverse diaphragms in the bridge decks in his discussion on a paper which appeared in the Transactions of the A.S. of C.E., vol. 114, 1949. Mr. Balog discussed the same problem in his recent article on "Grid Bridge Design" (Civil Engineering, January 1957, p. 40), arriving at the same conclusion as Mr. Lount in his paper, that the A.A.S.H.O. specifications should not be applied to interconnected bridge girders as they would give a false picture of the transverse load distribution.

The main advantage of the bridge decks consisting of several longitudinals

a. Proc. Paper 1303, July, 1957, by A. M. Lount.

1. Lecturer, Dept. of Civ. Eng., Imperial College, Univ. of London, London, England. In charge of a special graduate course on "Stress Analysis of Grids and Interconnected Bridge Girders".

interconnected with transverse diaphragms is their ability to distribute the concentrated loads applied directly upon a small portion of the structure, to all members of the grid. This decreases the high stresses in the directly loaded members and increases the stresses in the more distant members of the grid, thus achieving a fairly even stress distribution over the whole structure.

The determination of the stress distribution in the interconnected bridge systems seems to be rather difficult, at least for a designer tackling such a problem for the first time.

Bridge decks, analysed as grids, are in most cases highly redundant and this is certainly the main reason why many practising engineers prefer to analyse these structures by very approximate methods often neglecting completely the interconnection between the longitudinal girders and the transverse diaphragms.

As a rule, the precise analysis saves a certain amount of material, proving that in most members the real stresses are far below those calculated by approximate methods. It also shows that in some members the stresses can be severely underestimated by the approximate analysis.

Mr. Lount has to be congratulated on his most excellent paper on this important problem of the load distribution. The readers should be grateful for the clear way in which he points out the main difference existing between the grid bridges and bridges without transverse diaphragms in the case of overloading and reaching the elastic limit in some girders.

This is a very important point which is rarely stressed sufficiently strong by other authors discussing the advantages of the interconnected systems.

It is, however, a pity that Mr. Lount does not give any bibliography on this subject. The only reference, cited on page 1303-20 may easily convey a completely wrong impression upon many readers about the practicability of some relaxation methods. In the hope that American engineers would find useful in their studies such a bibliography on the stress analysis of interconnected bridge systems, the writer has appended a list of some papers published on this subject. It is not intended to be comprehensive but perhaps it will be welcome by some students of this problem.

Mr. Lount obviously does not know Leonhardt's or Homberg's books on the stress analysis of interconnected bridge systems as his paper leaves the reader under the impression that for moving loads it is always necessary to set up and solve a system of simultaneous equations which can be extremely sensitive. In Mr. Lount's experience a total of twelve decimal places were essential for the solution of his six simultaneous equations. Mr. Lount at several places states, and specifically emphasizes, his statement that it is only the advent of the electronic computer which made it possible to calculate from first principles the load distribution pattern in grid structures.

An engineer faced with a similar design problem of the stress analysis of interconnected bridge girders, after reading Mr. Lount's paper, might feel that without the use of electronic computers, the task of determining the stress distribution in grid systems is extremely difficult, if at all practicably possible.

The writer regrets to be unable to agree with all these points of view. For regular lay-outs of interconnected bridge systems and for the usual number of main and cross girders there are nowadays easily accessible books or papers which enable the designer to construct the influence surfaces of bending moments or shears for any point of such structures without even setting-

up of simultaneous equations. Typical examples of such works are the diagrams or tables prepared by Leonhardt, Homberg, Hendry and Jaeger, to mention only a few. They enable the designer to find the lateral load distribution coefficients for any stiffness factor, automatically taking into account the span of the bridge, the spacing of the main and cross girders, their second moments of area. Some of the above mentioned tables can be applied without any complications also to bridges in which the outer main girders differ in their dimensions from those of the inner ones.

The simplicity of these tables is obvious to anybody who at least once used them in practical application. Another important point is that for regular layout of bridges of the deck type in which the torsional rigidity is neglected (a case discussed by Mr. Lount in his example) even working from first principles, only in very rare cases one finishes with more than six simultaneous equations. Such a number of equations does not require the use of electronic computer. The solutions can be obtained in a very reasonable amount of time, even using the "old" orthodox methods. This is not only the writer's own opinion, it is shared by all his students, who, in their course on stress analysis of interconnected bridge girders, have to do several practical design problems on the load distribution over steel bridge decks. This is rather fortunate, as only very few engineers can avail themselves of the use of electronic computers.

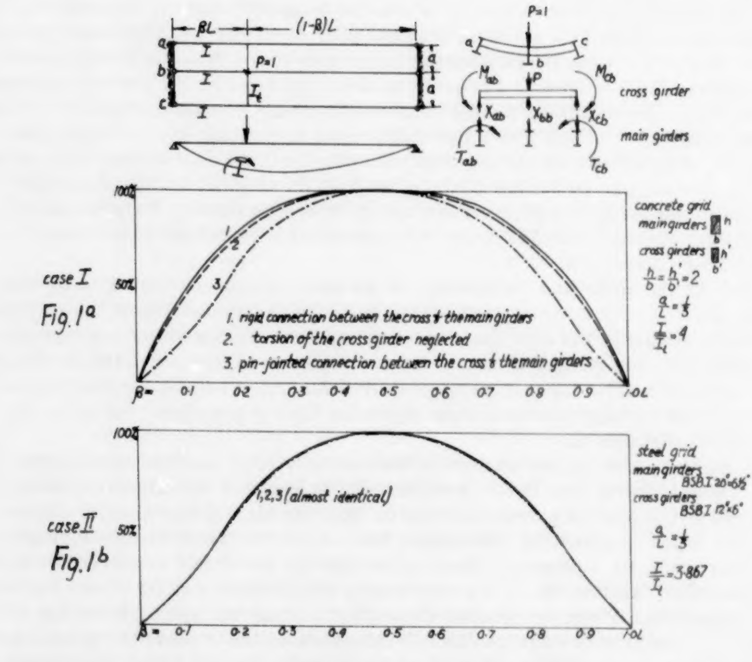
The writer finds that the method of the inverse matrix proves to be the most convenient for the analysis of grids. The determination of the influence surfaces requires the solution of several systems of simultaneous equations in which the only change is due to different positions of a unit load on the grid. The inverse matrix takes advantage of this fact as it can be applied to give the solution to many simultaneous equations having the same matrix A but different constants B.

A survey, done by the writer, of the recently built steel bridges of the deck type shows clearly that there is a tendency to restrict the number of main girders which only in exceptional cases exceeds six. It can also be shown that not much is gained by increasing the number of transverse diaphragms and that probably 3 adequate diaphragms are all which are required for good lateral load distribution. Very interesting conclusions can be drawn regarding the position of the transverse diaphragms along the span. In bridge design, it is customary to judge the effectiveness of the transverse diaphragm by its ability to distribute the load acting directly upon it between all main girders.

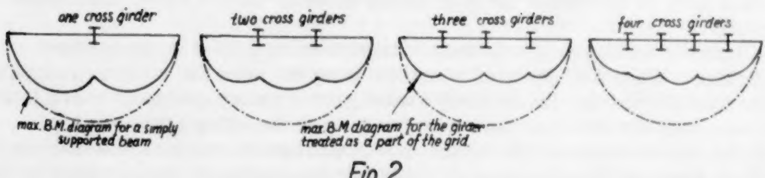
A transverse diaphragm is most effective when placed in the midspan of the bridge. When a diaphragm is moved along the span the lateral distribution of the load decreases, the directly loaded girder taking gradually more load and receiving a decreasing amount of help from the other girders.

If the effectiveness of the transverse diaphragm in the load distribution is 100% at midspan, the variation of the effectiveness can be represented by diagrams similar to those illustrating specific cases, in Fig. 1. The diagrams are the results of a general study (done by the writer) on the effect of rigidity of the connections and the variation in the number of transverse diaphragms upon the lateral load distribution in grid bridges. The study referred to was carried out under the assumption that the main girders were simply supported but were restrained against torsional twist at their ends.

Three curves are shown for each of the two cases illustrated in Fig. 1.



βL
(1-β)L
a
b
c
I
P=1
I_t
P=1
cross girder
main girders
M_{ab}
M_{cb}
X_{ab}
X_{bb}
X_{cb}
T_{ab}
T_{cb}
100%
50%
case I
Fig. 1a
1. rigid connection between the cross & the main girders
2. torsion of the cross girder neglected
3. pin-jointed connection between the cross & the main girders
β= 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0L
concrete grid
main girders
cross girders
$\frac{h}{b}=\frac{h'}{b'}=2$
$\frac{a}{L}=\frac{1}{3}$
$\frac{I}{I_t}=4$
case II
Fig. 1b
1,2,3 (almost identical)
steel grid
main girders BSB.I 20"×6½"
cross girders BSB.I 12"×6"
$\frac{a}{L}=\frac{1}{3}$
$\frac{I}{I_t}=3.867$



one cross girder
two cross girders
three cross girders
four cross girders
max. B.M. diagram for a simply supported beam
max B.M. diagram for the girder treated as a part of the grid

Fig. 2

Curve 1 obtained for a fully rigid connection between main and transverse girders. (i.e. allowing for rotation and twist of all members).

Curve 2 neglecting the influence of torsional rigidity of the cross girder; and

Curve 3 assuming a pin connection between the main and transverse girders.

Case I represents a typical reinforced concrete grid whose members are rectangular and for which $h/b = 2$. This ratio of the height of the member to its width is frequently used in practical design. The diagram shows that in reinforced concrete bridge grids there is a substantial difference in the behavior of the rigid and pin-jointed structure.

Case II illustrates a steel grid built with British Standard Steel Beams. It is obvious from this diagram that it is legitimate to analyse the structure as pin-connected, since the influence of rotation and twist of the longitudinal and transverse members are negligible.

Many experiments, carried out by the writer in the Department of Civil Engineering of the Imperial College, University of London, proved beyond any doubt that in steel grids, because of the very low torsional stiffness of the I-profiles, the influence of torsion rarely changes the stresses by more than a few percent. (This conclusion does not apply to steel girders of the box section for which the influence of torsional rigidity is very appreciable).

Fig. 1 shows also that if there is only one transverse diaphragm it is most effective when placed in the middle of the span.

In fact one transverse diaphragm at L/2 is equivalent to two diaphragms placed at L/3 and 2L/3. Similarly, three diaphragms at L/4, L/2, 3L/4, produce a diagram of maximum Bending Moments almost identical with that for diaphragms placed at L/5, 2L/5, 3L/5 and 4L/5. This is confirmed by the comparison of the diagrams of maximum Bending Moments obtained by the writer in 1949, for a steel grid bridge of 100' span, consisting of six main girders, spaced 11' apart, and for a varying number of the transverse diaphragms. In each case the diagrams of the maximum moments were obtained by means of influence surfaces which were drawn for various points along the span and determined analytically by using a precise analysis which took into account the real number of transverse diaphragms.

The inescapable conclusion is that, for a really efficient interconnected bridge system, an odd number of transverse diaphragms is required, with one at the center, which is most important for adequate lateral distribution.

However, going through the lay-out of grid system for the Big East River Bridge, the writer finds that Mr. Lount decided to use four diaphragms.

It is obvious to the writer that Mr. Lount certainly had some good reason for choosing an even number of transverse diaphragms for his design.

The writer would be obliged, however, if Mr. Lount could express his own opinion about the "oddness" or "evenness" of the number of transverse diaphragms.

The writer hastens to assure the author that his remarks are not intended to be destructive. He certainly agrees with Mr. Lount's remarks about the advantages resulting from the use of the electronic computers for very complicated bridge structures, but he feels that ordinary bridge designers need the assurance that even without using these computers, they are still in a

position to carry out grid analysis for normal bridges in a very reasonable amount of time.

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THOMAS D. Y. FOK,[1] A.M. ASCE.—The author has presented an excellent example in the application of the electronic computer to the solution of a practical structural problem. The basic approach to the solution of a problem in indeterminate structures is by use of the elastic deformation of the structure. In the case of a grid, interaction forces between the longitudinal members and the transverse diaphragms due to a unit load at an intersection point of the grid are solved by equalizing deformations of intersection points. Simultaneous equations are formed in terms of these unknown forces. The sensitiveness of these equations depends on the properties and arrangement of the members and the position of the loading. The loss of accuracy of the results is due to the inherent character of the computational process in which a large number of significant figures are lost by cross multiplication and subtraction of nearly equal values of the coefficients in the equations.

F. J. Murray has investigated the relationship between the number of significant figures in the coefficients and the correct figures of the solved unknowns in working with simultaneous equations. An approximate error formula is used to determine the probable error of the final results.[2] The writer has found that the formula is very helpful in determining the number of significant figures in the coefficients of the simultaneous equations.

In the formula the most probable error in the roots e_p is expressed in term of e_c, the error in the coefficients. If the number of the equations is n, then

$$e_p = n^4 e_c$$

By using this formula, the number of figures used in the coefficients are determined by the required accuracy of the roots.

The writer has been engaged in the design of a tied arch bridge in which a continuous rib is connected to a continuous stiffening truss by eight flexible hangers. This bridge is indeterminate to the ninth degree. Elastic equations are set up to solve for the unknowns—the hanger stresses and the horizontal

1. Design Engr., Richardson, Gordon and Associates, Pittsburgh, Pa.
2. Mario G. Salvadori: "The Mathematical Solution of Engineering Problems," Columbia University Press, New York, 1953, p. 128.

thrust in the rib. Eight significant figures are used for the coefficients with error of coefficient e_c equal to 0.00000001. The probable error of the roots will be

$$e_p = 9^4 \times 0.00000001 = 0.00006561$$

Since most of the roots are equal to 0.50000 ±. The roots are accurate up to about 1 out of 1,000 which is adequate for practical design. The writer has checked the results by back substitution into the equations. In most cases the error was found to be under the probable error as computed by use of the previously indicated formula.

Influence Lines Due to Moving Loads Located Between Two Girders

Since influence lines for the grid structure are determined by the unit moving load along each longitudinal girder only, influence lines for the loading position located between two girders can be determined by the following procedures.

1. Influence ordinates for this loading position can be computed by linear interpolation between influence ordinates along two adjacent girders. Influence ordinate

$$i = \frac{i_o + (i_1 - i_o)}{s} x$$

where i_o and i_1 are influence ordinates taken at a selected cross section along the longitudinal girders. (See Fig. 1_a.)

2. Approximate influence ordinate for a load between two girders also can be determined by using the influence ordinates for all the girders at a cross section taken through the point where a load is located. This can be accomplished by application of method of polynomial expansion. The computation of these influence ordinates can be incorporated into the program for the computer.

As indicated in Fig. 1_b, the influence ordinates obtained at the cross section selected are plotted for each girder. These ordinates are points on a curve that can be utilized in determining the influence ordinate at any position across the cross section selected by use of the following equation:

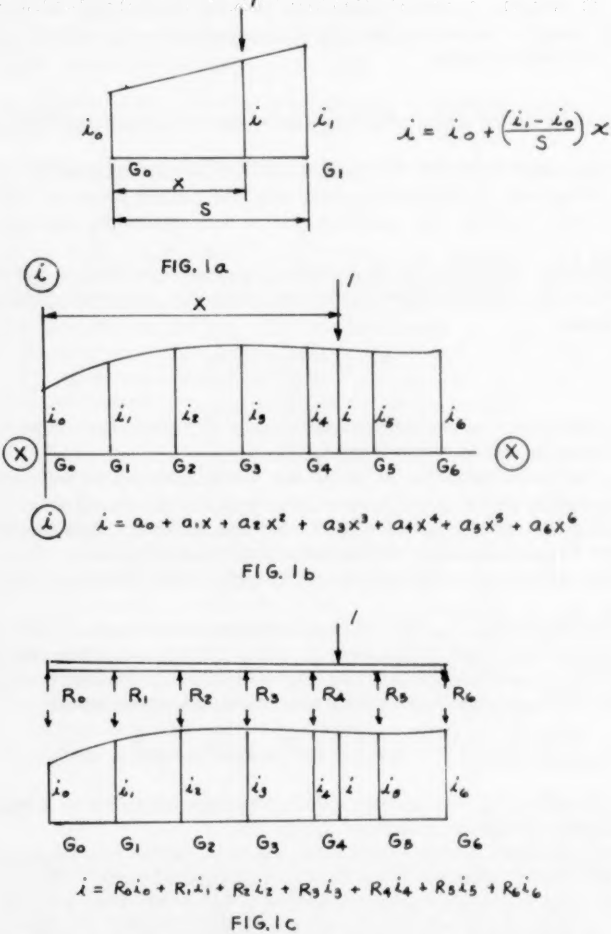
$$i = a_o + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 ,$$

in which the values a_o and a_6 represent constants obtained by a polynomial expansion using the given values of i_o and i^6.

3. If one assumes a continuous slab, the total of the reactions obtained at each girder can be substituted for the applied load of unity. The influence ordinate for any particular point located in a selected cross section of the structure is obtained by summation of the product of each girder reaction and its corresponding influence ordinate. The computation of these influence ordinates can also be included in the computer program. In Fig. 1_c, the influence ordinate

$$i = r_oi_o + r_1i_1 + r_2i_2 + r_3i_3 + r_4i_4 + r_5i_5 + r_6i_6,$$

where r_o to r_6 represent reactions on the girders, and i_o to i_6 are influence



DETERMINATION OF INFLUENCE ORDINATES DUE TO A LOAD LOCATED BETWEEN TWO LONGITUDINAL GIRDERS

ordinates at any selected cross section along the longitudinal girders.

The accuracy of the influence ordinates computed by each of the above methods can be determined by comparing the summation of all forces resulting from interaction between longitudinal girders and transverse diaphragms due to a unit load at a particular location in the structure. Since the summation of these forces should be equal to the unit load. The method which can produce values closest to unity is the most accurate. The author has presented diagrams of the maximum bending moment envelope due to live load. The manner of live load distribution between longitudinal girders that was used has not been presented. Possibly, one of the procedures demonstrated above has been employed. If some other method was used, its presentation would be of value and interest.

FATIGUE RESISTANCE OF PRESTRESSED CONCRETE BEAMS IN BENDING[a]

Discussion by P. W. Abeles

P. W. ABELES,[1] M. ASCE.—The authors say in their paper that the writer has mentioned in his paper (5) a ratio 1 of dynamic ultimate strength to static ultimate strength. This is based on a complete misunderstanding of what is said in the paper (5) and in the previous publication in the Journal of the Institution of Structural Engineers of October 1951. In both papers test results with regard to two composite, partially prestressed slabs were reported which were first loaded statically to such an extent that visible cracks developed, whereupon pulsating loads corresponding to the live-load range were applied. During this dynamic loading the cracks opened and closed 1 million times in slab 1 and 3 million times in slab 2. In the latter case the maximum load was increased for the 2nd and 3rd million of repetitions of pulsating loads at which also the range of loading varied from that of the 1st million repetitions. Notwithstanding the fatigue loading, static failure tests carried out for each of the two slabs showed that the ultimate static strength was not affected by the previous dynamic loading.

These results should obviously not be generalized, because they depended on the specific properties of the test specimens and on the range of loading together with the corresponding stresses. A ratio of 1 between dynamic and static ultimate strength, as mentioned by the authors, would hardly be feasible even under the most favorable conditions. In fact, any definite ratio of dynamic and static ultimate load is meaningless, unless especially qualified by the properties of the test specimens (percentage of reinforcement, magnitude of effective prestress, efficiency of bond and strength properties of the materials) and the loading (range and maximum) in connection with the corresponding stresses.

The writer would have liked to discuss in more detail the authors' valuable paper, but is unable to do so at the present moment. He would like to refer to his contribution to the "Symposium on Bond and Crack Formation" of RILEM in Stockholm, June 1957, in which 9 tests on prestressed concrete specimens are discussed based on the research carried out by the Chief, Civil Eng. Dept. of British Railways, Eastern Region, 1951-1957.

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a. Proc. Paper 1304, July, 1957, by C. E. Ekberg, Jr., R. E. Walther, and R. G. Slutter.

1. Asst. Civ. Engr., British Rys., Eastern Region, London, England.

DETERMINATION OF THE 0.02 mm FRACTION IN GRANULAR SOILS[a]

Discussion by Irving Sherman

IRVING SHERMAN,[1] A.M. ASCE.—The calculation of an average relation between the fraction passing the No. 200 sieve and the fraction finer than 0.02 mm is a useful method under certain conditions for reducing the number of hydrometer analyses which are necessary. Mr. Johnson is to be commended for his approach, which he has noted is valid only for similar soils.

However, such a correlation may suffer from the lack of a causal physical relationship between the two variables. If the fraction passing the No. 200 sieve is considered to be unity, the fraction finer than 0.02 mm may be anything between unity and zero.

If statistical correlations are to be used in an effort to reduce the number of hydrometer analyses, it seems preferable to base the correlations on soil properties which are physically more closely related to the soil fraction which is to be estimated. This writer previously presented[2] a description of such a method which is based on the relationship between surface area and absorbed moisture content of soil.

The use of statistical correlations to reduce the volume of laboratory analysis will necessarily cause some reduction of accuracy in the end result. If accuracy is important, such correlations may still be useful as tools for preliminary investigation and as means for selecting specific samples for detailed analysis.

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a. Proc. Paper 1309, July 1957, by R. W. Johnson.

1. Assoc. Civil Engr., Los Angeles County Flood Control District, Pasadena, Calif.

2. Sherman, Irving. A Rapid Substitute for Textural Analysis. Journal of Sedimentary Petrology, Vol. 21, No. 3, pp. 173-177, September 1951.

ALUMINUM APPLICATIONS FOR HIGHWAY BRIDGES[a]

Discussion by S. K. Ghaswala

S. K. GHASWALA,[1] A.M. ASCE.—The paper is a good generalized version of the potentialities of aluminium alloys in highway bridge construction.

The valuable properties of low density and good tensile strength have enabled these light alloys to be incorporated in bridges like the 55m span bridge at the Dusseldorf exhibition in Germany; the 60 ft. span, all welded aluminium foot bridge (the first of its kind) in South Wales; the conveyor bridge built in Iraq for transporting vehicles; the 26m span bridge built of 14S-T6 alloy in Japan; the bascule bridge of 115 ft. opening over the Tancarville canal in France; the 41.4 ft. long road bridge constructed over a canal near Szabadszallas, 50 miles South of Budapest; the 160 ft. long footbridge near Onex Golf Club, Geneva, and others.(1)

In India although aluminium has still not been used in any major bridge project, it was adopted for quite a similar purpose. In the construction of the 50-ton precast concrete beam bridge at Hardwar, North of Delhi, aluminium was used for the design of a special launching truss for placing the heavy precast girders in position. The bolted light alloy truss which was supplied by a U. K. firm was of triangular construction in cross section and its single-bottom boom was an extrusion of wine-glass shape. The truss weighed 5.75 tons having sides of 7.5 ft in panels of 7.25 ft. of N-type design. By the use of a temporary trestle and mast arrangement and taking one of the precast girders as a counterweight the truss is launched out from the shore side. When the other end is sited over a bridge pier, the trestles are adjusted to carry the ends of the truss and the main girders are slid out under this aluminium truss ready for being lowered in position.

Increasing use of aluminium in bridges will demand a deeper understanding of the basic principles of design, centered round such aspects as selection of the material and alloys used and the type and form of the superstructure; exact evaluation of loading and stresses; a complete knowledge of the best and most economic forms of construction technique; appraisal of surface treatment to give least maintenance and a harmonious incorporation of aesthetic value in the structure.(2, 3)

It is a well known fact that as the span of a bridge increases the dead weight of the bridge itself goes on increasing more rapidly until a stage is reached when the dead weight becomes so high that in that particular design or form of construction it becomes impossible to build that bridge. This is evident from the fact that for short spans plate girders are adopted but for longer spans trussed girders and arches are used. In the case of very long

a. Proc. Paper 1312, July, 1957, by J. M. Pickett.

1. Chartered Engineer, Bombay, India.

bridges, suspension structures are the only ones that can be adopted because of their comparatively low dead weight.

Since in such large structures the criterion of dead weight assumes significance, this writer is of the opinion that aluminium will in times to come, play a more important part in long span bridges than in short span structures as at present. Dr. Sutter and Marsh of the Aluminium Laboratories, Geneva[4, 5] have carried out some very interesting analysis on aluminium bridge spans and come to certain conclusions which, although they may differ for different conditions, nevertheless do give a very lucid picture of the basic facts. Thus a bridge in medium strength alloy of aluminium having an ultimate tensile strength of 18 tons/sq. inch, will become economical under a certain set of conditions in comparison with mild steel bridges from spans beyond 650 ft. for simple trusses; 1300 ft. for arch structures, and 3500 ft. for suspension bridges. If high strength aluminium alloys are used having U.T.S. of 28 tons/sq. inch, and they are compared with high tensible steel, then the economic limits of spans reduce further, being 350 ft. for simple trusses, 700 ft. for arches and 1800 ft. for suspension bridges.

Aluminium requires to be designed from first principles with a fuller understanding of its characteristics such as low elastic modulus, low density, a different type of stress-strain relationship as compared with steel, and the availability of very thin-skinned sections through the process of extrusion.

As this writer has stressed many times[2] aluminium has come into the structural field via the aircraft industry and as such should give a striking indication of its beneficial and progressive influence, especially in bridge engineering, provided that it is not sidetracked into the familiar pattern of imitation of traditional materials. Herein the important considerations are in the fields of elastic and inelastic instability and their attendant modes of failure; fatigue and creep and their relation with notch effects; and the plastic behaviour of light alloy structures and their repercussion on Poisson's ratio and design trends including the concept of Safety Factor.[6 to 12]

Although these aspects are being slowly understood and investigated yet considerable practical experience remains to be gained before the first half-mile long arch bridge or a mile-long suspension bridge in aluminium can be designed or erected.

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SELECTION OF THE GROSS SECTION FOR A COMPOSITE T-BEAM[a]

Discussion by A. Zaslavsky

A. ZASLAVSKY.[1]—The influence of creep, etc., is seen to be quite small in the normal cases when the dead load of the second stage (M_{DLc}) is small in comparison with the live load (M_{LL}). Combining M_{DLc} and M_{LL} into a single moment $M_c = M_{DLc} + M_{LL}$ acting upon the composite section with the modular ratio n would make the calculation a little shorter and still accurate enough. Referring to authors' Example No. 2, the stresses thus calculated are (M_c = 144 + 876 = 1020 k-ft):

$$fc = \frac{1020 \times 12}{2059 \times 10} = 0.59 > 0.57$$

$$f_t = \frac{1020 \times 12}{2939} + 13.11 = 4.16 + 13.11 = 17.27 \text{ ksi} < 18.03 \text{ ksi}$$

$$f_b = \frac{1020 \times 12}{1163} + 7.46 = 10.52 + 7.46 = 17.98 \text{ ksi} < 18.12 \text{ ksi}$$

The authors' paper is based on the conventional elastic method of design by admissible stresses. In the composite beams under consideration the design is governed by the extreme fibre stresses f_t and f_b of the steel beam, the stress f_c in the concrete plate remaining low. The section is thus designed so as to have the admissible steel stresses produced by the bending moment M_{tot} of the total load.

Although this elastic method is also adopted in Europe (including Eastern Europe), the writer feels that some modification is called for by allowing for the non-proportional relationship between stress and load (or moment).

The writer suggests that the design should be based not on the admissible stress f_{adm} but rather on the maximum elastic moment M_{YP} producing the yield stress f_{YP} in the extreme fibres of the steel beam. The admissible moment M^*_{tot} would then equal M_{YP}/SF (SF = safety factor) which is not identical with the moment M_{tot} producing f_{adm} directly. The same problem generally arises in structures having no stress-load proportionality in the elastic range.[2]

Considering A7 steel with a minimum yield stress of 33 ksi and an admissible stress of 18 ksi, the safety (or load) factor is 33/18 = 1.83. Referring

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a. Proc. Paper 1313, July, 1957, by R. S. Fountain and I. M. Viest.

1. Senior Lecturer, Technion, Israel Inst. of Technology, Haifa, Israel.

2. Zaslavsky, A., "Safety Factor in Structures in the Elastic Range without Stress-Load Proportionality," Bull. Res. Counc. of Israel, vol. 6C, No. 2 (in press).

again to authors' Example No. 2, the stress-moment relationship is plotted in the writer's Fig. in full lines. It is represented by a broken straight line with its segments corresponding to the three loading stages. Producing the lines of the final loading stage (LL) up to the point of appearance of f_{YP} at the bottom fibre we arrive at M_{YP} = 3008 k-ft. Using authors' safety factor 33.00/18.12 = 1.82 the admissible total moment should equal M^*_{tot} = 3008/1.82 = 1652 k-ft > M_{tot} = 1564 k-ft.

The proposed method leads to more economical sections especially when, for some reason, it is impossible to utilize f_{adm} simultaneously in both extreme fibres. Assume, for instance, the same section as before but with M_{DLs} = 650 k-ft, M_{DLc} = 150 k-ft. We ask for the admissible moment of the live loads. Combining $M_c = M_{DLc} + M_{LL}$ the f/M relationship is represented by dashed lines parallel to the corresponding full ones. It is readily found that M_c = 1225 k-ft; therefore:

$$M_{LL} = 1225 - (650 + 150) = 425 \text{ k-ft.}$$

On the other hand:

$$M_{YP} = 2985 \text{ k-ft; } M^*_{tot} = 2985/1.83 = 1630 \text{ k-ft}$$

$$M^*_{LL} = 1630 - (650 + 150) = 830 \text{ k-ft} > M_{LL} = 425 \text{ k-ft.}$$

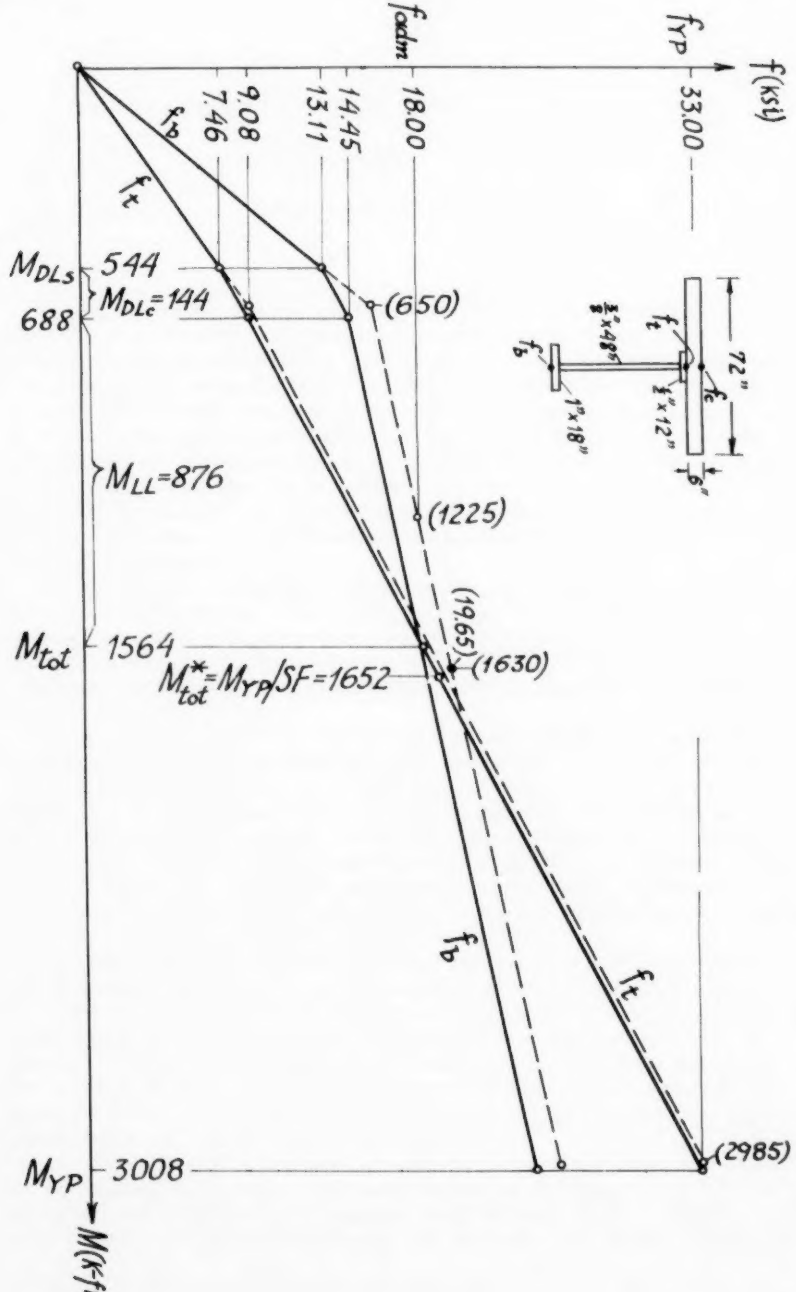
While M_{tot} is governed by the top fibre, M^*_{tot} (through M_{YP}) is governed by the bottom fibre.

The working stress produced by M^*_{tot} = 1630 k-ft equals 19.65 ksi. For such a reliable material as steel, even higher working stresses should raise no objection, a load factor of 1.83 having been provided for against the appearance of the yield stress.

It should be pointed out that there may be cases where the direct conventional method would lead to a reduced load factor ("concave" f/M lines). In this case the proposed method would demand a working stress lower than the "admissible" one.

The complex problem of establishing a rational safety factor is not discussed, but it would be interesting to have authors' opinion at least on the question of different factors for dead and live loads.

Lastly, the writer would like to suggest that in designing composite sections (especially in buildings) the plastic strength reserve could also be taken advantage of. It would be logical to do so in view of the growing acceptance of ultimate load design. In this case, the carrying capacity of the section would be largely independent of the loading stages and the non-proportional relationship.


f(ksi)
f_{adm}
f_{YP}
33.00
18.00
14.45
13.11
9.08
7.46
f_b
f_t
M_{DLs} 544
M_{DLc}=144
688
(650)
M_{LL}=876
(1225)
(19.65)
M_{tot} 1564
$M^*_{tot}=M_{YP}/SF=1652$
(1630)
M_{YP} 3008
(2985)
M(k-ft)
72"
1"x18"
f_c

LOAD FACTORS FOR PRESTRESSED CONCRETE BRIDGES[a]

Discussion by E. Neil W. Lane

E. NEIL W. LANE,[1] A.M. ASCE.—Particularly in the structural field, it is desirable to know what one is doing as compared to what one thinks (or just agrees) is being accomplished. There is nothing worthy in keeping the left hand uninformed, or partially so, as compared to the right, where matters of strength and economy are paramount. Practicing engineers are highly indebted in general to our teaching profession (for the testing and interpretations) and manufacturers and governmental agencies (for the materials and money) by means of which research is enabled to advance our science. So, our thanks here go to Professor Lin, who has provided us with a brief, yet complete, minima of discussion as to where our design thoughts meet the ultimate in the realm of practicality.

One can hardly discuss load factors for bridges without some natural overlapping of ideas relating to buildings, piers and the like. Moreover, it is necessary to delve rather carefully into the properties and differences of materials, and the procedures of design and construction, to say little of many complications and a number of unknowns, prior to an adequate consideration of load factors.

Primarily this discussion should apply to over ninety percent of our highway bridges that are in suitable use over spans of a hundred feet or less and carry the equivalent of the A.A.S.H.O. H-15 loading or better with a service expectancy of 30-50 years at the rate of 500-800 vehicles per hour, and normally stress the combined members in the 80 percent range, thereby allowing about a twenty-five percent overload every 20-30 repetitions. This would be in the 1.0-1.5 million cycle dynamic load range. Under the best of conditions, except by specification, actual loads are seldom known within ten percent, and this is more often believed to be 20%.

By means of a 90% of median minimal-steel-stress specification and an 80% definition for allowable concrete with good inspection, the producing industry is provided with a reasonable working range in which to provide the consumer a satisfactory product at a competitive cost with minimum rejection. These are minimum uncontrollable bands of doubt for materials whose range is about as well known for dynamic as static conditions. The false economy of poor inspection at the expense of heavy members, except for small quantities, can be easily visualized.

The use of legalized loadings or load limit signs on bridges and buildings or pier aprons is, at best, a palliative protecting the designer or building department, but scarcely much satisfaction to the unknowing workman or military enlisted man under order to get the job done. This is particularly true

a. Proc. Paper 1315, July, 1957, by T. Y. Lin.

1. Lodi, California.

where the order giver fails to realize (or cares less) that giving carries a responsibility where the recipient is less able in one or several ways.

Many a hide is saved by the hundred percent impact factor and seventy percent moisture condition factor used in the design of a timber pier (or trestle) and the simple span construction provides a shock dampening effect at the extreme far end of a gamut of conditions represented by the plant fabricated and field joined dock platform of combined tee and box girder construction with prestressing to consider in three planes over the supports. A ship's first acquaintance with such a structure usually commands more respect the next time, but in most cases hull damage can be overcome by better guard rail and buffering devices, as well as ship control.

By contrast, the bridge has about the same chronological life, usually a much longer cyclical life (probably double), a much greater range of live load to dead (though with a much smaller impact factor), and small lateral loads of wind and traction as compared to the bumping of ships from wind, waves and landing.

The overturning of hard-tired forklift trucks provides a policing action against overloads that a longshoreman respects and is much more effective than the state patrol can hope to be against a small percentage of the public. However, this can be small consolation to the driver that suffers a break caused by the previous passage of an overload. The public is always against the fellow who was there when it apparently happened.

These are instances of coming load factor considerations that will require attention as the comparatively new product (prestressed concrete) enters an old field such as bridge, pier or building construction.

Where accuracy of alignment and minimum vibration are determining items in the design of jet testing trackways, railroad bridges and machinery foundations, the relatively greater flexibility of prestressed concrete can be a distinct disadvantage and force the load factors so high as to be of little concern.

Somewhat along the same line, the proper structural prestressing force often causes excessive dead load camber for the best sightliness along the exposed edges of some of our parking garages and bridges. In flat roofed building construction, this large range of alignment with light loads sometimes causes conflicts of drainage, esthetics and economy that precludes much concern over an inadequate load factor. The excessive working of joints in service can cause maintenance considerations to supercede load requirements.

In as much as no adequate analysis and design would be complete without elastic as well as ultimate considerations of the conditions affecting safety and serviceability, it is to be expected that much loose interchange of remarks concerning load factors and safety factors will arise, but as long as it is clearly realized that they apply in an interrelated manner to a similar end to be gained by two completely different methods, a little harm perhaps can lead to a better understanding that is sought.

Plant and construction site matters of mishandling of members and improper steel placement, it is felt can only be handled by proper specifications and inspection. The engineer can hardly be held responsible for crude or unusual operations of others beyond the pale of standard practice or reasonable expectancy with good judgment. This should not be a cause for increasing the load factor at the owner's expense.

Study of the change in load factors as one proceeds from the comparatively thin shelled box (via various tees and channels) to the solid rectangle with the prestressing force closely localized shows a 20-50% improvement in the efficiency with an increasing imbedment-depth ratio as well as a decreasing compressive stress ratio.

The distribution of the prestressing steel over a cross section (such as a box, channel or tee) can be the cause of considerable trouble in the end block zone due to improper distribution of direct stresses causing stress reversals in superfluous material and diagonal tension (requiring additional non-prestressed rebars) elsewhere. These members of wide faces and thin sections require a careful consideration of the shear lag and combined stresses to be successful and all too often one sees them run into an abrupt change of shape at the end block. Most adequate end blocks seem to require a length of roughly 1.5-2.0 times the beam depth for 55-65% prestress ratios and lengths of about 3.0 depths for up to a maximum recommended prestress at 85% of ultimate strength. The usual structural civil engineer can profit by more acquaintance with the stress problems of the machine and equipment designing mechanical engineer. Where the grouting and anchorage of the cables or tendons is confined to the end zone and in relatively long and wide members these troubles are accentuated. From these considerations, it is easily seen how shape and proportion ordinarily considered most efficient for loading can actually have detrimental features when used for prestressing and require higher load factors than otherwise thought necessary. This is really unfortunate when the real cause is not the mid-span section with properly distributed stresses, but the anchorage zone so often of unsatisfactory design.

The greatest change of angle in the bent up portion of the prestressing cables usually occurs near the sharpest change of section from midspan shape to end block or perhaps a couple of depths nearer midspan. The position and amount of bent up steel, and its relative stress again are factors causing a variation in the requirements for the design load if one is to maintain fairly consistent performance and reserve strength at a minimum for economy.

Where continuity is desired without the development of cracking, to establish bond in ordinary reinforcing bars, such as the field joining of precast, prestressed portions of rigid frames, attention is called to the successful use of the wedge-jack as shown by the Japanese and Russian engineers at the recent World Conference For Prestressed Concrete at San Francisco. The Tokyo version was particularly successful in earthquake resistant buildings with closely confined working conditions. There is also a procedure for inserting a precast block or plate to prestress ordinary welded bars lapping a field joint of precast members that utilizes the dead weight of the parts in lieu of the jacks.

In lift slab construction, an important point in favor of prestressing, is the increase in allowable punching shear with tendons in top of the slab each way at the columns. With thin slabs and comparatively thick casings, careless strand placement at or below mid depth can be just as injurious as helpful at the top. Pullout failures have occurred in some instances.

A similar adverse result was obtained by the Structural Design Section of the Department of Architecture for the State of California when a floor slab reinforcement failure was induced by the Structural Engineers as they attempted to have a bar stressed when lain with inadequate imbedment, or anchorage on a right angle turn with about a three foot radius. While such a detail could be corrected by sufficient cover in some cases, it could also have

been obviated either by balancing the split out vectors of the curved bar bearing on the concrete by cross typing along the curve to a similar longitudinal on the opposite side of the thin slab, or, radial hairpins could have been applied on the convex side of the curve keeping the outbound tendencies within due limits for diagonal tension caused by this Poisson effect.

Such facts indicate the need in a load factor for a consideration of lack of experience, ignorance, carelessness and/or a lack of control or authority in some instances quite outside of our available knowledge of materials and procedures or methods of design and construction.

The form of the load factor formula is a statement of a recognized acceptable loading condition for the design of a particular type of structure for a particular use. It is a derivative of the earlier methods of elastic analysis. Like Lloyd's manual for structural design of ships, it has given satisfactory experience before and it is believed has every reasonable expectation of doing so again in the added light of the latest knowledge available to us.

From the standpoint of the transition of the safety factor of elastic analysis to the load factor of ultimate analysis for concrete, it is easily seen that $2 \times 0.45\ f'_c t\ 20\% = 1.2\ f'_c$. For steel, we obtain the familiar plastic design coefficients in $2f_s$ (at 18.0 ksi) $(1.57 \times 1.15) = 2 \times 1.83$ ksi $= 2fy = f'_s = 66.0$ ksi, the test range median with ten percent clearance above the rejection limit. These compare with the ACI-ASCE recommendation of $0.8\ (2D + 3L)$ for bridges showing an allowance for load ratios and other contingencies.

Were one to follow the parable of the 'blind men and the elephant' and examine the above from the standpoint of the live to dead load ratio predominantly, a quite different formula would evolve, perhaps as follows:

$$F_1 D / \sqrt[a]{\frac{L}{D}} + F_2 D \sqrt[b]{\frac{L}{D}}$$

providing an increase in the dead load factor with a declining live to dead load ratio. For the range recommended by the BPR, ACI and ASCE, we find $F_1 = 1.6$, $F_2 = 2.3$, $a = 3$, $b = 5$ and $4.0 \geq L/D \geq 1.0$.

Should the load factor formula be looked upon from the standpoint of dynamic loading only, Professor C. E. Ekberg of Lehigh University, has provided the profession with a combination of modified presentations of the Goodman-Johnson diagram for steel and concrete in relation to the stress-moment relations in a bridge girder, that would lead one to a load factor formula of the following type:

$$\text{For steel, } F_1\ (D + 2L) / \left(\frac{\text{Allowable Stress Range}}{\text{Actual Stress Range}}\right) - n$$

And for concrete, the tensile portion of the stress range could be ignored without appreciable effect on the factor.

This all points to the fact that there is no simple panacea for a lack of experience and ability. After all of these empiricisms and conjectures, experienced analysis and judgment is still the best bet in the absence of specific tests. Professor Lin is due our thanks for adding to this in a very capable manner.

SYNOPSIS OF FIRST PROGRESS REPORT OF COMMITTEE ON FACTORS OF SAFETY[a]

Discussion by Milos Vorlicek and Jan Suchy

MILOS VORLICEK and JAN SUCHY.[1]—It is a difficult problem to estimate the factor of safety; it is first necessary to take into consideration all factors influencing its value. We have analyzed, therefore, some mechanical properties of building materials. In this discussion we wish to show the results achieved by the evaluation of homogeneity of concrete and of reinforcing steel.

The traced property of a material, termed x, can be characterized by a probability curve p(x). We look for the minimum value x_{min} of the property. We determine this lower limit x_{min} in such a way that from the whole probability area, limited by the curve p(x) and the horizontal axis, we neglect a small part P. In other words, we choose a probability P of occurrence of a smaller value that x_{min}. We choose such a small probability P (usually 10^{-2} up to 10^{-6}), so that it is practically quite certain that there will occur no smaller value than the minimum value x_{min}. In order to characterize the homogeneity of material, we determine the ratio of the minimum value x_{min} to mean x_o; this ratio is called the coefficient of homogeneity h_x:

$$h_x = \frac{x_{min}}{x_o}$$

For estimation of the values x_{min} and h_x it is necessary to know the distribution p(x).

The Gauss-Laplace law cannot be applied for the distribution of the compressive strength of concrete in many cases; we must look for another more suitable curve. We tried to find the distribution of the strength of concrete on a theoretical basis. The strength of concrete depends on many influences, quality and ratio of components of concrete, the way of mixing, curing, etc. Therefore we used some known relations, according to which the strength of concrete is a function of these influences. We carried out four independent derivations based on formulae of Bolomey, Feret, Graf, and Bendel. These relations consider only some principal quantities which influence the strength of concrete; we extended them by an arbitrary finite number of further secondary factors. In all four cases we assessed that the distribution of compressive strength of concrete can be illustrated by Pearson's curve of type III. The equation of this curve, in the simplest shape, is the following

a. Proc. Paper 1316, July, 1957, by Oliver G. Julian.

1. Czechoslovak Academy of Sciences, Praha, Czechoslovakia.

$$p(t) = \frac{a^{a^2} e^{-a^2}}{\Gamma(a^2)} (a + t)^{a^2-1} e^{-at};$$

$\Gamma(a^2)$ is the gamma-function; the constant a is given by the equation

$$a = \frac{2}{g_c},$$

where g_c is the coefficient of skewness. The strength of concrete f_c and the variable t are bounded by the relation

$$f_c = f_{co} + t.s_c ,$$

where f_{co} indicates the average, s_c the standard deviation of the strength of concrete.

This theoretical result was certified by a sufficient number of tests of the strength of concrete accomplished in Czechoslovakia as well as abroad and we determined that Pearson's curve can be conveniently applied.

Pearson's curve of type III shows the distribution with various skewness; this is of great advantage. If the coefficient of skewness is zero, Pearson's curve changes to the well-known Gauss-Laplace curve. The coefficient of skewness will be estimated from obtained results of tests independently, because it determines (together with the average and with the standard deviation) the shape of distribution whereas, for example, the log-normal distribution is the coefficient of skewness the function of average and of standard deviation only. Using probability P of occurrence of a smaller value than $f_{\widehat{c,\ min}}$, we determine the minimum strength of concrete $f_{\widehat{c,\ min}}$ from the relation

$$f_{c,\ min} = f_{co} - t_{min}.s_c ;$$

the value t_{min} is taken from Pearson's curve of type III. The following table shows the values t_{min} computed for various probabilities P from 10^{-2} to 10^{-6} and for the coefficient of skewness g_c from 0,0 to 1,1:

Coefficient of skewness	$P = 10^{-2}$	$P = 10^{-3}$	$P = 10^{-4}$	$P = 10^{-5}$	$P = 10^{-6}$
0,0	2,33	3,09	3,72	4,26	4,75
0,1	2,25	2,95	3,51	3,99	4,42
0,2	2,18	2,81	3,30	3,70	4,08
0,3	2,10	2,67	3,10	3,44	3,75
0,4	2,03	2,53	2,90	3,18	3,43
0,5	1,95	2,40	2,71	2,94	3,13
0,6	1,88	2,27	2,53	2,71	2,85
0,7	1,81	2,14	2,35	2,49	2,60
0,8	1,73	2,02	2,18	2,29	2,36
0,9	1,66	1,90	2,03	2,11	2,15
1,0	1,59	1,79	1,88	1,94	1,97
1,1	1,52	1,68	1,75	1,78	1,80

We determine the coefficient of homogeneity of concrete h_c from the expression

$$h_c = 1 - t_{min} \frac{s_c}{f_{co}} .$$

In practice, the known method being used, it is quite sufficient to compute from test results the average and the standard deviation and to determine the coefficient of skewness g_c according to following expression

$$g_c = \frac{1}{n.s_c^3} \sum_{i=1}^{n} (f_{ci} - f_{co})^3 ,$$

where n is the total number of specimens tested and f_{ci} indicates the individual test results.

Fig. 1 shows data resulting from 471 tests of the strength of concrete, corresponding to Pearson's curve of type III. In this case we choose the probability $P = 10^{-3}$, the minimum strength is $f_{c,\ min} = 212\ kg/cm^2$ and the obtained coefficient of homogeneity is $h_c = 0{,}59$.

We deduced from the theoretical basis a method enabling us to determine the coefficient of homogeneity of concrete in structures from the non-destructive hardness tests with the adapted Poldi-hammer/K. Waitzmann: The hardness tests as a mean for determining mechanical properties of building materials, Acta technica No. 1-1956, Praha/. We get convenient results already from a very small number of tests.

It was impossible by theoretical means to fix the probability curve of the reinforcing steel, because we are not yet perfectly acquainted with the factors influencing the yield point depending on the making of steel. Therefore we were compelled to choose a convenient theoretical curve by means of tests. It was evident that the Gauss-Laplace curve cannot be used because the test results of yield point showed a distinct positive skewness and therefore we applied Pearson's curve of type III, too.

The applicability of this curve was ascertained by aid of Pearson's criterion x^2; the agreement was found to be very good in all cases. In Fig. 2 we show as an example the distribution of yield point of the mild reinforcing steel having diameter 16 - 24 mm. We tested 307 specimens. The minimum yield point for this steel /using $P = 10^{-3}$/ is 32,3 kg/mm^2, the coefficient of homogeneity is 0,78.

To determine the coefficient of homogeneity of each material, supposing that the Pearson's curve of type III can be applied, we made a simple nomogram (see Fig. 3). Hence for given coefficients of variation and of skewness we fix the coefficient of homogeneity from the nomogram /the nomogram is construed for the probability $P = 10^{-3}$.

ERNEST BASLER.[1]—The writer would like to point out two reasons which in his opinion make this report particularly valuable: first; numerical examples are included which illustrate the large variation in the strength properties of our building materials. The samples used in Fig. 2, 3 and 5 show the statistical distribution of the yield strength of structural and reinforcing steel, and that of compression strength of concrete. Although the distribution

1. Design Engr., Anderson, Birkeland and Anderson, 1123 Port of Tacoma Road, Tacoma 2, Wash.

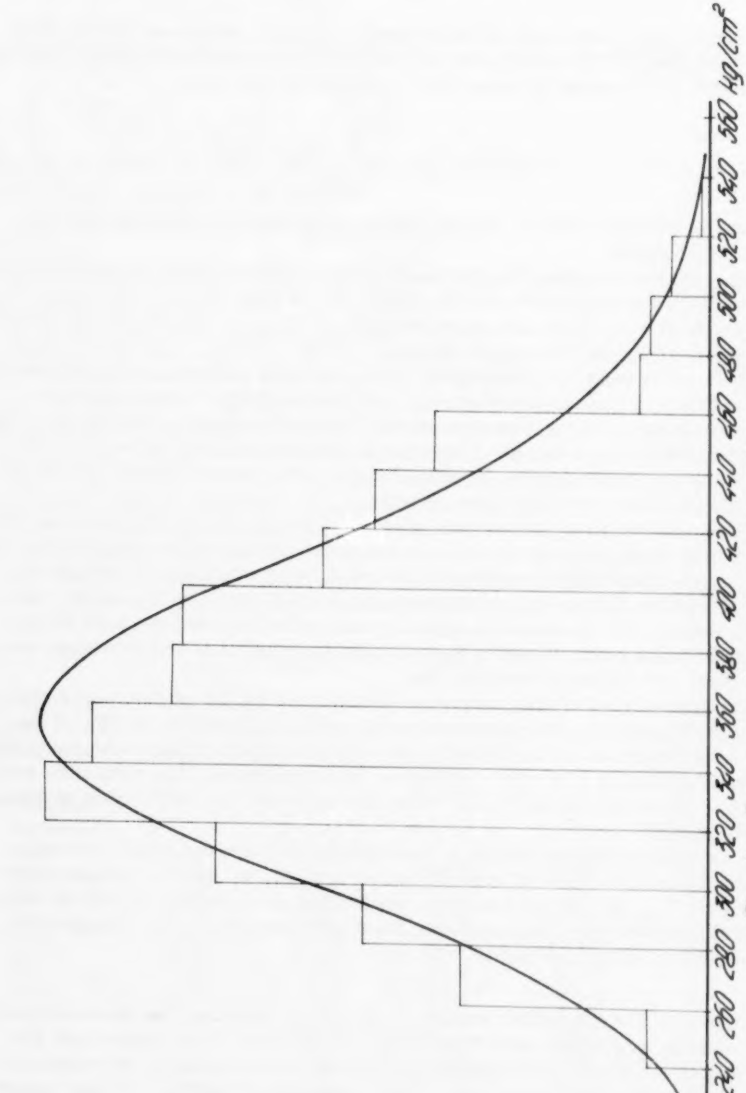


Fig. 1.- Compressive strength of concrete.

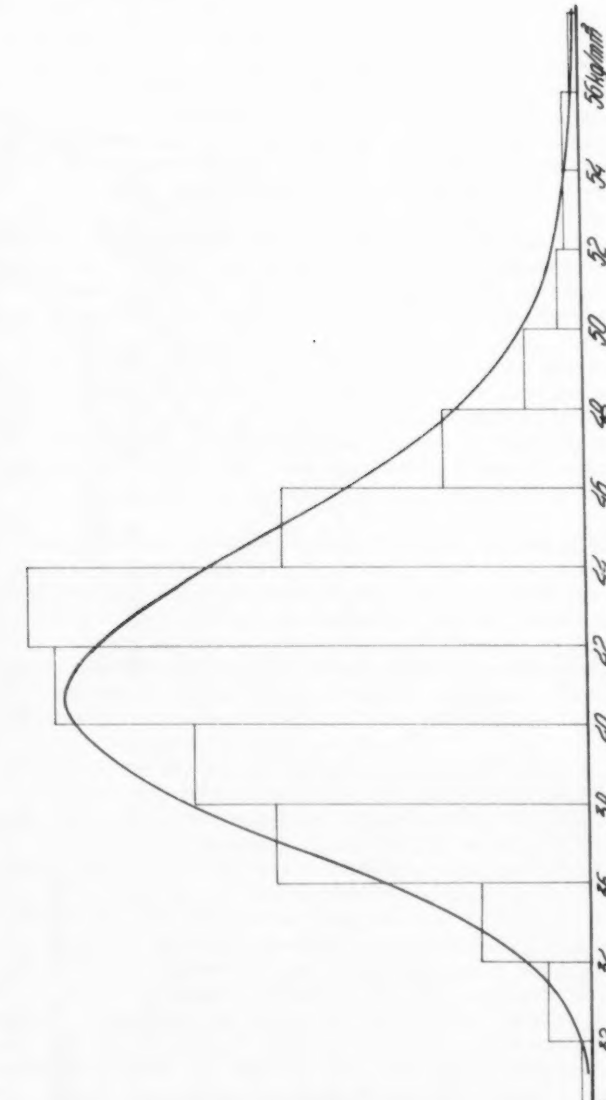


Fig. 2 - Yield point of reinforcing steel.

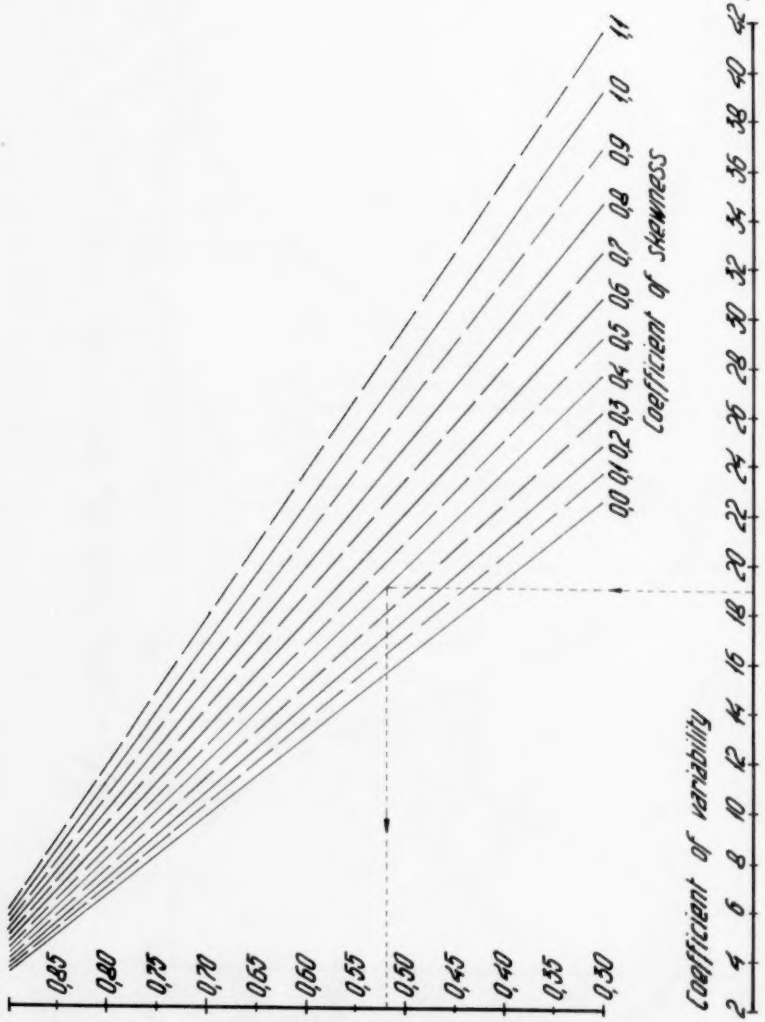


Fig. 3 - Nomogram for determining coefficient of homogeneity.

appears to be typical, by comparison with data published recently in Germany,(1) it is desirable that more samples be taken particularly at the construction site. Second; the Committee stresses the joint effect of uncertainties in external loadings and internal strength of materials. The importance of a correlation of these two random phenomena cannot be emphasized enough, and a reason why this has been neglected so long is that there are usually two entirely different groups of specialists who formulate or influence design loads and minimum strength values: There are national, state or local government officials advised by meteorologists, seismologists and others, on one side, while on the other side are manufacturers of building materials and specialists in the technology of materials. Clearly, it is the design engineer who stands between the two groups and who has to bring the two sources of information together by means of the structural analysis. If, however, the design engineer asks for information from the neighboring branches of engineering in a greatly simplified form (e.g. furnish a single constant for what in reality is a whole distribution function), the two neighboring groups are forced to round their figures off and put a certain safety margin into their specifications on account of the later oversimplified treatment. If, however, a certain safety in the load assumptions is already included in form of an unlikely or infrequent occurrence, the same is done in the field of material technology, and the design engineer superposes his own safety factor, then it is likely in some cases that the end result is unreasonably safe.

In an effort, therefore, to achieve maximum economy and at the same time adequate safety the design engineer has to insist that all the information on the strength properties of materials and the anticipated loads on the structure are given to him in a completely unbiased form and as realistic as possible. However, in most cases, this is only possible by means of statistics. The design engineer subsequently ought to be in a position to read this statistical information and derive results and conclusions therefrom. The Committee on factors of safety has made an important contribution toward this ultimate goal.

To include in every structural design an analysis of the probability of failure may be too time-consuming and may not always be justified. However, the writer sees three immediate applications of these theories:

1. All codes dealing with design loads and minimum strength properties of materials should base their recommendations on a specified (constant) probability of occurrence. As long as this is not done almost any load can be claimed to be a justified design load. Just how arbitrary many of the specified design loads are can be illustrated by a comparison of recommendations for the same type of loadings by independent authorities. Comparing for instance the American Standard Building Code Requirements for Minimum Design Loads in Buildings and other Structures (1955) with the corresponding Swiss code (Normen fuer die Belastungsannahmen von Bauten, 1956), it is found that the two codes may disagree up to 95% either way for the same type of loading. Herein the comparison is made only for loading groups which by their very nature should not differ much from country to country, such as apartments, classrooms, stairs, habitable attics, office-buildings, lobbies etc. This, together with the fact that most design loads are rounded off to rather substantial increments, indicates clearly that the corresponding probabilities of occurrence of the various loadings within a single code must vary greatly. The above comparison refers to "man-made" loadings; for all loads

influenced by nature, e.g. loads caused by wind, snow, rainfall, temperature, earthquake, etc., it has been proven repeatedly by meteorologists and seismologists that they are essentially randomly distributed, and here it is even more evident that a single load coefficient is a very poor substitute unless it is correlated with corresponding probabilities of occurrence.

In specifying design loads, more thought should be given to the combinations of some loads. If the occurrence of two types of loads are independent of each other, as wind and earthquake are, and the frequency distribution of each is known, then it is relatively easy to derive a frequency distribution of the joint occurrence. It would however be of high importance to know the correlation between partially dependent loadings. Wind and snow, wind and temperature extremes, and snow and temperature may not be quite independent, and observed correlation factors would help to construct more realistic probabilities of occurrence for any such load combination.

2. There is a vast field of qualitative applications (judgment) which can be used to great advantage in the everyday work of the design engineer. Consider for instance an underreinforced concrete member subject to bending in which there is a choice of reinforcing with one large bar or several small bars. Each bar has a possible deviation below the specified minimum yield strength, but the probability of any one bar being weak at a particular section is greater than that of a group of bars. If the allowable working stress is established by applying a factor of safety to the specified minimum yield, it would then be proper to use a higher factor of safety in the design employing one large bar than in the design with several small bars. On the other hand if using only one factor of safety, one may say that the design with several small bars is the better one.

The above example may be generalized as follows: Whenever a multiple of structural elements is used and their relation to the structure as a whole is such that the weakest element causes unserviceability or failure of the total structure (e.g. weakest link of a chain) a somewhat bigger factor of safety with respect to the average strength is warranted as compared with a case where the capacity of all elements has to be exhausted before the capacity of the total structure is reached (e.g. the chain links are placed parallel to each other and stressed together).

3. For unusual structures it may be well justified, and sometimes necessary, to carry out a complete analysis of the probability of failure or impaired usefulness. Some of the most wonderful structures would probably never have been built if the engineers would not apply some principles of these theories. For such an example reference is made to the recently completed 700-foot high TV tower near Stuttgart in Germany.(2) This concrete tower, which also contains a restaurant some 500 feet above ground, is a free-standing vertical cantilever. The worst loading condition which has to be assumed for design and stability is not a steady lateral wind force alone, but a series of wind gusts which cause the highly elastic and slender tower to vibrate. The natural frequency is about one cycle in 6 seconds. Since the damping coefficient is small, the wind gusts of observed intensity could cause the tower to collapse provided they are assumed to reoccur frequently enough, on the total projected area of the tower, and always in phase with the natural vibration. However, meteorological observations have shown that wind gusts are essentially randomly distributed in time, location and intensity. Furthermore, studies also show that any sequence of gusts which would lead to overturning, within the anticipated lifetime of the structure, is extremely im-

probable. But it cannot be said that the probability is zero. Any engineer who would deny the assumption of randomness and just add up all the conservative assumptions without checking the likelihood of a joint event would rarely be in a position to build such an impressive structure.

The writer would like to know whether a systematic study has ever been made to find out which distribution functions are most appropriate for structural engineering purposes. Although there are many functions which satisfy the theoretical requirements of a frequency function, one is confined to a few which for practical reasons combine the following requirements: The function used should be easy to fit to a given histogram, it should also have convenient mathematical properties, and, if possible, tables of properties for various parameters together with the cumulative frequency function (integral) should be available. Since almost all histograms describing strength properties of materials are somewhat skewed towards the lower values, and also have a clearly defined minimum value and ill defined upper limit, it seems that the Gamma Function would be an excellent substitute, and it would probably also be suitable for some extrapolations to extreme values. The Gamma distribution is defined as:

$$f(x) = kx^{\alpha} e^{-\frac{x}{\beta}} \qquad (1)$$

where k is an integration constant and α and β are parameters.(3) Tables of the Gamma Function were published in 1922 by the Office of Biometrica under the editorship of Karl Pearson. In the form shown above the function is defined in the interval $o < x < \infty$. However, by replacing x with x - a, the function can be shifted and any lower limit a can be assumed. Fig. A shows the Gamma Function f(x) for values $\alpha > 1$.

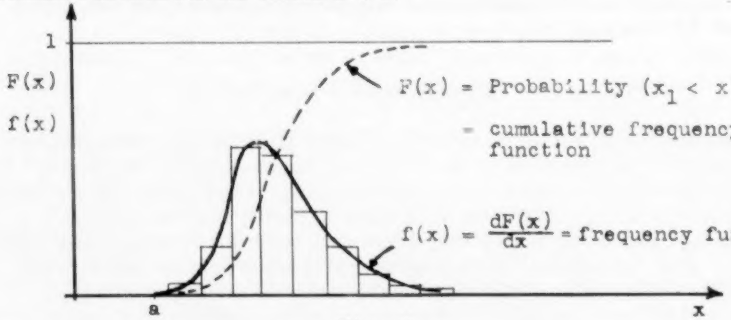


Fig. A

The same figure also shows and defines the cumulative frequency function F(x) which is identical with the probability that an event x_1 is smaller than the stated value x. This definition, applied to all values of x, including those above the mean value, enables a somewhat easier and more rigorous treatment of a probability investigation in that the cumulative frequency function can be obtained by a continuous graphical or numerical integration of the histogram. Conversely, having F(x), the frequency function f(x) is obtained and expressed by the first derivative of F(x). The writer also found that the less common presentation of the cumulative frequency function which the Committee used in Fig. 1 to 6 of its report has initially mislead many people because of its similarity to the frequency function.

The Committee on Factors of Safety must be congratulated on this first report. Many problems are clearly formulated by now and would be ripe for more extensive quantitative investigations. However, the labor of collecting and processing pertinent data is large, and it is hoped that the Committee can enlist the support of other interested groups that have the facilities and personnel to undertake research in this field.

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3. Cramer, H., Mathematical Methods of Statistics, Princeton University Press 1954.

RENE E. WALTHER.[1]—More than the half of the papers published in Vol. 83, ST 4 of the Journal of the Structural Division of the ASCE Proceedings deal directly or indirectly with the problem of structural safety. As may be noted, there are distinct differences between the basic concepts in several of these papers. Some comments about this broad subject are presented herein as a discussion to Mr. Julian's valuable paper, firstly because it pertains to the majority of questions raised and secondly because it embraces the aspects of structural safety most generally and may thus serve to reflect the mentioned differences.

Probability—Factor of Safety—Load Factors

The assessment of safety factors on the basis of probability of failure or unserviceability, as proposed by the author, is indeed truly rational and at least from a theoretical standpoint very convincing. Even though the concept in its present form will, in the writer's opinion, hardly ever be applicable to general design problems, it may well serve as a guide for drafting codes and especially as an "eyeopener" for a proper appraisal of the true and complex nature of structural safety. Notwithstanding these restricting remarks, the writer wholeheartedly welcomes the efforts of the Committee on Factors of Safety and he endorses the plea for further studies especially with respect to problems of fatigue under traffic load.

Mr. Julian states: "The factor of safety and the factor of serviceability can be considered as 'load factor' by which the mean of the design load-effect is multiplied to equal the mean calculated resistance." This being true, it must be said, however, that the practical use of load factors deviates considerably from the author's definition in that separate load factors for dead and live load were introduced (see for example the paper by T. Y. Lin(1) and that by Michell and Borrmann).(2) Now in the light of probability considerations separate load factors become meaningless. Given for example the

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standard deviation of the resistance and of the dead and live load, then an infinite number of load factor combinations pertain to one and the same failure probability. Or on the other hand, if the separate load factors are stipulated, then the failure probability is not uniform for structures of different sizes even though their standard deviations may be equal. Obviously factors of safety in the sense proposed by Mr. Julian and load factors as widely advocated belong to two entirely different concepts and it seems high time to make a drastic decision which way should be followed. In the writer's opinion, a rational approach as suggested by the Committee on Factors of Safety deserves definite preference.

Beside its lack of rationality, which is an inexhaustible source for unqualified arguments, the concept of separate load factors leads often to questionable contentions for which the papers mentioned above may be cited. Michell-Borrmann(2) note for example: "Why not specify an ultimate live load only? And as for the dead load, if maximum weights of material are specified and future surfacing computed as additional uniform load, there seems no reason to use a load factor (1.2 to 1.5) for dead load except to take care of approximations and carelessness in design computations." This is just one example of the widespread opinion, favored by the load factor concept, that uncertainties stem only from the load assumptions. Furthermore, the writer is somewhat concerned about possible liberal interpretations of statements like: "So far as the overload capacity is concerned, the use of a proper load factor definitely insures the ability of the structure to carry overloads." (Lin, Ref 1)

Probability Distribution

In spite of the undeniable advantages of the notion of safety proposed by Mr. Julian, there remain a number of complex questions to be answered, the foremost of which is probably the choice of the theoretical probability distribution. The author has certainly to be complimented for openly presenting the excessive discrepancy between factors of safety based on Gauss-Laplace normal distribution and those based on log-normal distribution. Mr. Julian's reasoning for giving preference to the log-normal distribution does, however, not guarantee that the determining extreme values are accurately appraised. It must be strongly questioned whether sufficient data will ever be available for such an appraisal. We thus return to the contention indicated in the beginning of this discussion that the merit of the whole concept lies in guiding the engineering judgment rather than in disclosing numerical values. After all, the selection of the probability of failure or unserviceability is ultimately also a matter of engineering judgment.

Assessment of Standard Deviations

The standard deviation of resistance, however, involved its derivation may be, does not cause any notional difficulty, whereas the definition of the standard deviation of the load-effect needs some clarification. Caution should be exercised in interpreting traffic observations, because an accidental maximum load must be considered as a non-random event, which cannot be extrapolated from time dependent observations. In other words, the mean value of the frequency curve of long time traffic observations cannot be taken as the

mean value of design load-effect. The latter should thus be defined as the mean value of probable maximum load effect, the evaluation of which may well be based on statistical methods. In the case of highway bridges, for example, it seems reasonable to assume that the maximum load effect occurs when the bridge is fully loaded (bumper to bumper). The mean value of design load and the corresponding standard deviation can thus be obtained by simply observing the relative frequency of occurrence of specified vehicle types.

The suggestion by the author to employ engineering judgment for the estimation of the maximum load-effect and to consider the probability of occurrence as unity, is certainly very helpful and might clear the way for an expedient acceptance of the proposed concept.

As for fatigue problems, there remains a good deal of work to be done. Mean values and standard deviations of load-effect and resistance for fatigue require further precise definitions.

In conclusion, the writer wishes to compliment Mr. Julian and the Committee on Factors of Safety for clarifying and advancing a rational concept of structural safety. The synopsis constitutes an important step towards the advocated educational effort within the profession, which in the writer's opinion, is the major merit of such probability considerations.

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VIBRATION SUSCEPTIBILITIES OF VARIOUS HIGHWAY BRIDGE TYPES[a]

Discussion by Yeshayahu Etkin

YESHAYAHU ETKIN,[1] A.M. ASCE.—Mr. Oehler is to be congratulated for his informative report. Reports of this kind lead to a better general understanding of the resonant behavior of structures. However, there is one point that cannot be left without comment.

On page 12 the author stated that for simple span bridges the theoretical fundamental natural frequency was calculated with the equation

$$f = \frac{\pi}{2L^2} \frac{gEI}{W} \tag{1}$$

In order to derive this equation mathematically it is assumed that deflections, or amplitudes, are directly proportional to the external exciting forces. In other words, the spring coefficient, "k", in the general differential equation of motion is a constant. When

$$\frac{md^2y}{dt^2} + c\frac{dy}{dt} + ky = f(t) \tag{2}$$

This assumption is valid as long as the material obeys Hooke's Law. However for non-linear materials the spring coefficient, "k", is no longer a constant and the assumption is no longer valid. Hence it is necessary to determine what effect the non-linearity of the material has upon the calculated fundamental natural frequency of the structure. In other words any value of E, and sometimes I, substituted in Equation (1) must be carefully examined.

On questioning any assumed EI values it is necessary to remember that the possibilities of getting good correlation between a 'true' calculated and an observed value of the resonant frequency are limited. One possibility is to develop, mathematically, an equation which will yield the fundamental natural frequency while considering the variations of the spring coefficient, "k", under dynamic loading. Another possibility is to develop an empirical expression which will yield the 'true' resonant frequency. The mathematical approach was proved to be extremely difficult and was successful only when "k" was expressed as a very simple function. The development of an empirical expression is dependent upon extensive experimental data which is not available.

However, Mr. Oehler's seventeen test results show an unusual agreement of the calculated and experimental values of fundamental natural frequency (an average of less than one percent error). This indicates that the conventional usage of Equation (1) is possible if adequate E values are substituted

a. Proc. Paper 1318, July, 1957, by LeRoy T. Oehler.

1. Structural Designer, Moran, Proctor, Mueser and Rutledge, N. Y., N. Y.

in it. It will be most instructive for future investigators if Mr. Oehler can explain how he arrived at his assumed E values.

Moreover, the I value in Equation (1) need not be a constant. In the case of a composite design (i.e. composite action between steel beams and concrete deck) insufficient knowledge of the modulus of elasticity of concrete influences the modular ratio "n" and therefore, the calculated I value.(1) For reinforced concrete structures, where the beam is assumed to be partially cracked, the magnitude of error in the moment of inertia, I, is even greater. Obviously, the accuracy of the fundamental natural frequency, as calculated from Equation (1), is dependent upon the accuracy of the two variables E and I. Under dynamic conditions the variable flexural rigidity, EI, of the beam depends not only upon the quality of the material,(2) but also to a great extent on the magnitude and duration of the external exciting forces.(3, 4)

The writer's experience has shown that even with pretensioned prestressed concrete beams, where I was constant, an E derived from a tangent to a Load/Deflection curve caused a 10-15 percent lower calculated natural frequency.(5) Experiments demonstrated that 'plastic' deformations introduced most of the discrepancy in the evaluation of E_{static} (= under a static load) and $E_{dynamic}$ (= under dynamic load).(6) Therefore, a test in which the 'plastic' deformation is a minimum would yield a good value of E to be substituted in Equation (1). Experiments showed that for resonance computations of prestressed concrete beams and slabs, usage of the ultrasonic pulse method to evaluate $E_{dynamic}$ yielded calculated fundamental natural frequencies which corresponded very well with the experimental values.(5) However if a direct static test is more desirable than the ultrasonic pulse method, Whittaker's formula(7) may be used to obtain an approximate value of $E_{dynamic}$:

$$E_{dynamic} = (0.95\ E_{static} + 1.27)(x\ 10^6\ \text{psi}) \qquad (3)$$

Whether $E_{dynamic}$ of concrete is determined by ultrasonic pulse method, induced vibrations,(8) or any other way, the most accurate values of E, and I, which are substituted in Equation (1), should be carefully worked out and presented. Such a presentation would eventually clarify the influence that non-linearity, magnitude and duration of external forces, and properties of the material employed has on the dynamic behavior of structures.

It will also be of interest to know if the author's assumed values of modular ratio, E_{static} and $E_{dynamic}$ were considered to be constant for all the tested bridges, irrespective of their age, type of structural cross-section and type of bridge design.

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5. loc. cit. page 194.

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APPLICATION OF AASHO SPECIFICATIONS TO BRIDGE DESIGN[a]

Discussion by Herbert A. Sawyer, Jr.

HERBERT A. SAWYER, JR.,[1] M. ASCE.—In the comments of the authors on the use of Appendix B of the AASHO Bridge Specifications, no mention was made of the error which may cause permissable stresses calculated by this appendix to be as much as fifty percent over their correct value. This error and its correction were called to the attention of the profession in "Civil Engineering," V. 24, N. 4 (April 1954), p. 73, and copies of a more complete discussion of the error were distributed to those engineers most directly interested in the matter at about the same time, so it is presumed that the Seventh Edition of the specifications now in preparation will incorporate this correction.

The error referred to is in the definition of α. In Young's presentation of the original theory (9), α was correctly defined as the ratio of the total smaller to the total large end-eccentricity. In Appendix B, the allowance for equivalent eccentricity from probable column imperfections is $0.25r^2/c$. Adding the known end-eccentricities of e_s and e_g to this value to obtain total end-eccentricities:

$$\alpha = \frac{e_s + \frac{0.25r^2}{c}}{e_g + \frac{0.25r^2}{c}} \quad \text{or} \quad \alpha = \frac{\frac{e_s c}{r^2} + 0.25}{\frac{e_g c}{r^2} + 0.25} \qquad (1)$$

Appendix B of the Sixth Edition (1953) is thus incorrect in defining α as e_s/e_g. In Eq. (1), the eccentricities e_g and $0.25r^2/c$ must always be of positive sign, and e_s is positive if on the same side of the column as e_g and negative if on the opposite side. Thus, it is readily seen that the error (the difference between the two values of α) increases from zero to significance both as the ratio of e_s to e_g decreases from +1 to -1 and as the ratio of e_g to $0.25r^2/c$ approaches zero.

An example producing the maximum magnitude of this error would be a column with e_g and e_s equal, of opposite sign, and very small with respect to $0.25r^2/c$. For such a column with pinned ends and $1/r = 118$, the allowable average stress, using Eq. (1) with Appendix B, would be 10,000 psi, while the allowable average stress using the incorrect definition would be 15,000 psi. If, for an otherwise identical column, the small values of e_g and e_s are neglected, Appendix B states that α shall be taken as equal to +1, which value yields an average allowable stress of 10,000 psi. Thus, only with the use of

a. Proc. Paper 1320, July, 1957, by Eric L. Erickson and Neil Van Eenam.
1. Prof. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

Eq. (1) is the design continuity which must exist between these two almost identical columns maintained, and these examples tend to verify Eq. (1).

Correction of this error in Appendix B simply involves the substitution of Eq. (1), together with a note regarding sign convention, for the present, incorrect definition of α. Also, it is clear that this substitution will eliminate the need for both the sentence following Eq. (B) and the last paragraph on page 289 of the Sixth Edition. If these changes are made, all remaining formulas and all graphs of Appendix B may be used in their present form.

DOUGLAS T. WRIGHT.[1]—It is always of considerable interest and value to the profession to have described some of the background to those codes and specifications which are used widely. Certainly the AASHO Bridge Specifications are widely used, without the United States as well as within.

A number of features in the AASHO Specification have concerned the writer, and it may not be unreasonable at this time to pose some questions which relate rather more to general features than to specific details.

First, in the assignment of working stresses it always seems unreasonable, however frequently it occurs, that the same values are specified for live and dead loading. Since it is patently obvious that all those effects which must be covered by "factors of safety" for live loads do not apply to dead loads, we must in our present practice either be wasting material or neglecting useful strength. Although this neglected strength can be of great value when overload capacity is desired,—if then some use is made of rational procedures and higher levels are permitted for dead load stresses, it turns out that the margin of extra strength thus provided varies from bridge to bridge—indicating in turn that our usual practice does not provide uniformly safe structures. Most curiously, an earlier AASHO bridge specification, in 1931, limited live load stresses in reinforcing steel to 16,000 psi, while allowing dead load stresses as high as 24,000 psi.

The other point which seems of most concern is that there seems to be a certain unreality in the method of describing design loadings. Several alternates (4 altogether in the case of the new loadings for bridges on the Interstate System) are set, with the most critical to govern. Notwithstanding the trouble and time spent in considering different cases, such things as the fact that the concentrated load to be taken with the distributed lane loading varies in magnitude depending on the function being studied show that the design loadings are set up not so much to approximate actual vehicle effects as to produce certain desired results. This is also revealed in the way other loads must be used for long span bridges.(1) Though these procedures may be necessary to give required proportions, the same purpose would perhaps be better served by dispensing with the myth of "truck" loads and alternate conditions, and designing instead for specified axle loads and maximum moments and shears simply tabulated for various loaded lengths.

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MICHAEL E. FIORE,[1] M. ASCE and THOMAS R. KUESEL,[2] A.M. ASCE.—
Authoritative discussions of the background of specification clauses, and of the intentions of the persons composing specifications, are all too rarely published. All engineers who work with the AASHO Specifications for Highway Bridges should be grateful to Messrs. Erickson and Van Eenam for their discussion and for the bibliography and references that they have included.

In the hope that further public discussion will clarify the application of certain clauses concerning which questions have been raised in the course of the writers' practice, the following comments are offered:

Section 2 - Loads

The specifications concerning impact, as given in Article 3.2.12, do not clearly indicate whether or not impact is to be added to the live load bending stresses in the top struts of concrete framed piers. This type of construction is quite commonly used for expressway bridges, and since the number of stringers resting on the top strut generally exceeds the number of columns, there are substantial live load stresses in the top struts. It would seem that some impact should be included here, but that the percentage should be less than for superstructure members. Live load stresses in the columns and footings are generally small, so that little error can result from the neglect of impact in these portions of the pier. Some clarification of the intent of the specifications with respect to impact in this type of structure would be welcome.

Section 4 - Unit Stresses, Pile Loads, and Bearing Power of Soils

The fifth paragraph notes that Article 3.4.8 covers the unit stresses in structural nickel and structural silicon steel. It is the writers' understanding that the Specifications are intended to apply to bridges with spans of up to 300 to 400 feet, as stated in the Introduction. It would be a very unusual highway bridge with a span of less than 400 feet that would require the use of nickel steel. Modern high-strength alloy steels, such as the T-1 steel used in the new Carquinez Straits Bridge in California, or the special alloy steel developed for the new Mississippi River cantilever bridge at New Orleans, would seem to have replaced nickel steel, as a practical matter.

In similar fashion, silicon steel, although widely used in the past, is today being replaced by the so-called "low-alloy steels" covered under ASTM Specification A-242. Manufacture of silicon steel has always been difficult in that a large portion of the material coming through the rolls bears surface scars. These cause outright rejection of some of the material, and the remainder must be repaired by welding, in accordance with ASTM Specification A-94. Some engineers have refused to permit the surface conditioning of silicon steel by welding, and in these cases satisfactory material has been difficult and expensive to obtain. One of the advantages of the modern low-alloy steels is that the surface defects are much less widespread and more easily

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repaired. It should be noted that the principal manufacturers of steel do not make an alloy conforming exactly to the chemical composition and physical properties of ASTM Specification A-242, but that a number of proprietary alloy formulations approximate the specification requirements closely, and are usually satisfactory for structural purposes.

Since the composition of alloy steels used for bridges seems to be changing rapidly, and since the use of alloy steels is largely confined to longer spans, the writers would like to suggest that the AASHO Specifications might profitably include a general discussion of the factors to be considered in the use of alloy steels for long spans, such as is included in Part 1, Section "C" of the Specifications for Steel Railway Bridges of the American Railway Engineering Association.

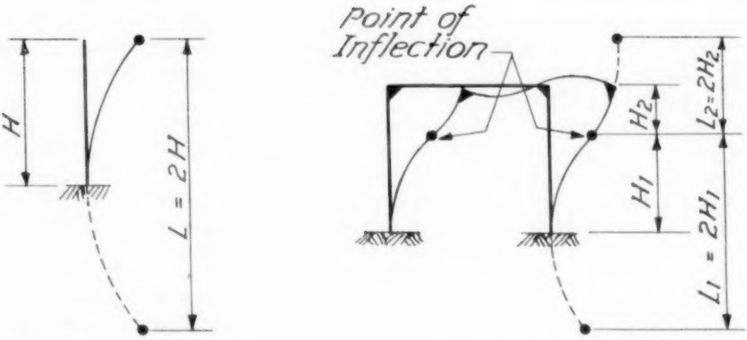
It would also be valuable to have a discussion of live loadings and unit stresses for long-span bridges. Although the design of long spans usually involves particular considerations that can be resolved only by the judgment of the engineer and is not amenable to rigid specification provisions, an authoritative discussion to serve as a guide in this matter could be quite useful. The writers know of at least one 700-foot span, which was financed by revenue bonds, and which was designed for H20-S16 live loading throughout, not for technical reasons, but simply because it was necessary or expedient to assure the underwriters that the bridge could carry "the heaviest standard loading of the American Association of State Highway Officials". Some statement in the AASHO Specifications concerning modification of live loading for long spans would have permitted the engineers to make a more economical design of the main truss members.

Incidentally, there is no statement in Section 4 of the Specifications that the allowable unit stresses for ASTM-A373 Structural Steel for Welding should be the same as those given for ASTM-A7 Structural Carbon Steel.

Section 6 - Structural Steel Design

The authors' discussion of unit stresses in steel columns subjected to combined compression and bending refers to Appendix "B" of the AASHO Specifications. It is not generally appreciated that the formulas and charts of Appendix "B" were developed on the assumption that there is no relative transverse displacement between the two ends of the column. In many cases involving combined compression and bending this assumption is not true—for instance, for portal frames of through trusses, for rigid frames of all types, or for cantilever columns fixed at the bottom and free to move at the top. The information given in Appendix "B" may be used for these cases if proper allowance is made for the length "L". To obtain an equivalent length of column for determination of allowable unit stresses based on consideration of buckling, twice the length from the inflection point to the point of fixity should be used, as shown in Fig. 1. For a free-standing cantilever column, the equivalent length should be twice the height of the column.

This converts the problem to an equivalent pin-ended column, for which the allowable unit stresses are given in Appendix "B", by Formula "A" and by the charts for $\alpha = +1$. However, it should be noted that the charts are drawn with the scale of L/r values based on the total length of the member, whereas the allowable stresses plotted on the charts are based on a buckling length of 75% of the total length of the member for riveted end connections, or 87.5%



Free-Standing Cantilever Column

Rigid Frame

L = equivalent column length for buckling.

L = 87.5 % of ℓ for Charts for Pin-ended Columns.

L = 75 % of ℓ for Charts for Riveted-ended Columns.

FIG. 1

Column Lengths for Appendix "B", AASHO Specifications.

$$f_p = \frac{f_b}{2} + \sqrt{\left(\frac{f_b}{2}\right)^2 + \left(f_v\right)^2}$$

of the total length of the member for pinned end connections. In order to use the charts directly, it is therefore necessary to divide the equivalent length of column (that is, twice the distance from the inflection point to the fixed point) by 0.75 or by 0.875 before entering the appropriate chart of Appendix "B".

For tapered columns, the radius of gyration for cases involving transverse displacement of the column should be taken at the point of fixity of the column. As noted in the footnote to Appendix "B", it may be necessary to check the column in the direction perpendicular to the plane of bending as a concentrically loaded long column, if the value of L/r is greater in this direction. For tapered columns, it may also be necessary to check the short-column stress at points of smaller cross-section.

The above discussion of column lengths applies with equal force to the design of concrete columns and frames. The design specifications of Articles 3.4.11 and 3.7.10 do not refer to this matter specifically.

There are several other points in Section 6 worthy of further comment. In discussing Article 3.6.75, concerning thickness of web plates for continuous beams and girders, it should be noted that the unit stress specifications of Articles 3.4.2 and 3.4.7 limit the allowable diagonal tension at sections where maximum shear and bending occur simultaneously. This diagonal tension is to be computed as the principal stress at the root of the flange of a rolled or built-up welded section, or at the lower gage line of the flange angle of a built-up riveted section, according to the standard formula for principal stress:

$$f_p = \frac{f_b}{2} + \sqrt{\left(\frac{f_b}{2}\right)^2 + (f_v)^2}$$

where f_p is the principal stress, f_b is the unit stress in bending, and f_v is the unit stress in shear. If this limit is observed, it is not clear why the minimum thickness of the web plate should be arbitrarily increased 25% over intermediate supports, as the authors state.

Article 3.6.80 covers the spacing of transverse intermediate stiffeners. It has long been noted that the AASHO Specification on this matter is more conservative than that of the American Railway Engineering Association. In cases where the spacing of stiffeners has become a critical item in the design, because of architectural or other considerations, the writers have referred to the rational method of stiffener spacing outlined by Mr. Otis E. Hovey in the March 12, 1931, issue of the "Engineering News-Record". This method is based on the work of Timoshenko, and in general gives slightly larger stiffener spacing than either the AASHO or AREA formulas. Use of this rational formula may be justified on the basis of the paragraph concerning rational design analysis that appears at the head of Division III of the AASHO Specifications.

The authors' discussion of welded highway bridge design is especially valuable. It may be worth emphasizing that the maximum section of a welded beam or girder is determined by the basic unit stresses given in Section 3.4 of the AASHO Specifications. The location of cover plate cut-offs or other changes in flange section, and the sizes of all welds, must be determined on the basis of the more severely limited unit stresses of the American Welding Society Specifications.

There is no special comment in the AASHO Specifications concerning welding of alloy steels. Although many alloy steels are weldable, the variations

in alloy composition and welding technique required are probably too great to permit a discussion of this matter in standard specifications. There should however, be some mention that welding of alloy steels is not to be undertaken without special consideration of the electrodes and techniques required for the specific alloys and conditions to be encountered.

Section 9 - Composite Beams

The Appendix to the authors' paper includes a complete revised specification for the design of composite beams, in Article T-14 (56). Under this heading, the second paragraph of Article 3.9.4 is concerned with the design of continuous composite members. Careful consideration of continuous composite construction by engineers who have attempted to design and construct such structures has brought out a number of questions that are not discussed in the authors' paper. The writers believe that satisfactory answers to the following points should be developed, and reduced to clear and concise specification clauses, in order to avoid confusion on this subject.

1. The dead load stresses in a continuous composite beam depend on the sequence of pouring the concrete slab. The writers believe it is basically a bad procedure to employ a construction in which the unit stresses in the completed structure depend on construction procedures, particularly if the design engineer does not have supervision over the construction. Some safeguards must be set up for this matter.

2. The determination of the elastic properties of a continuous composite beam is a complex question on which very little useful data is available. One approach would be to assume that the concrete slab contributes fully to the moment of inertia of the composite section throughout both positive and negative moment areas. Mr. John Sherman, M. ASCE, in Volume 119 (1954) of the ASCE Transactions (pp. 810 & ff.), has discussed some of the difficulties involved in this assumption, and indicated that it may involve errors in design moments of as much as 25%. The proposed specification clause quoted in the authors' paper appears to imply that the portions of the concrete slab that are in tension should be neglected in computing elastic properties, although this is not specifically stated. This approach requires the designer to compute the locations of the points of contraflexure by trial for any given fixed loading (since the elastic properties of the continuous beam and the location of the points of contraflexure are inter-dependent). For moving live loading, these two items cannot be clearly determined with any confidence, and the designer is quickly forced into assumptions of limiting cases. Further complications arise when consideration is given to the questions of the effect of bond between the beam and slab in negative moment areas, the redistribution of elastic properties resulting from creep of the concrete, and whether the effective slab width should be the same in areas of positive and negative moments.

3. It should be evident from the above that a conscientious designer, in the face of these uncertainties, must design for the worst conditions arising from limiting assumptions for the variables or unknowns involved. This not only takes more time and increases the cost of the design, but also increases the cost of construction, since extra material must be included to cover all of the limiting cases.

4. In the absence of guidance from design specifications concerning these

matters, individual engineers will make differing assumptions according to their preference and experience. This will result in different solutions to the same theoretical problem in different design offices and under the jurisdiction of various public agencies, which is clearly not desirable.

5. There is some evidence that continuous composite construction tends to increase the prevalence of cracks in concrete bridge decks, in the areas of negative moment. Particularly in cold climates, where salts and chemicals are used to remove ice in winter, this will aggravate the problems of deterioration and maintenance of bridge decks.

Although continuous composite design may be permitted by the specifications, the writers believe that many engineers will continue to avoid this type of construction because of the uncertainties in the analysis, the greatly increased cost of design required compared with the small economies in cost of construction to be realized, and the possible adverse effects on the integrity of the bridge deck. Acceptance of continuous composite design cannot be enforced unless and until a satisfactory technical basis for it is developed.

In conclusion, the writers should like to reiterate their appreciation of this valuable interpretation of the AASHO Specifications, and trust that Messrs. Erickson and Van Eenam will comment further in their closing discussion on the points raised herein.

Note: Two small typographical errors in the paper as published in the July, 1957, Journal of the Structural Division should be noted. On page 1320-5, in the fourth line from the bottom, the formula should read "S/(4.0 + 0.25S)", instead of "S/(4.0 + 2.5S)". On page 1320-6, under Section 4, the eighth line should read "Group III = Group I + LF + 30% W + WL" instead of "Group III = Group I + LF + 30% + WL".

HENRI PERRIN.[1]—The AASHO Specifications are designed to cover many types of bridges and therefore may not be ideal for each particular case. However, the writer feels that some points need clarification.

The design wind velocity as defined in the paper was fixed as 100 Mph and it should be understood that a gust factor has already been included, thus giving the following relationships:

For trusses and arches: $F = .00256 \times 2.90\ (F.V)^2 = 75\ \text{lbs/ft}^2$
For girders and beams: $F = .00256 \times 1.95\ (F.V)^2 = 50\ \text{lbs/ft}^2$

where: 2,90 and 1,95 are shape factors for trusses and girders respectively. F is a gust factor, generally(1) chosen as equal to 1.30 and V the wind velocity in miles per hour.

For F = 1.30, V becomes 77 Mph and corresponds to a pressure of 35 lbs/ft^2 as indicated in the wind pressure map given in the revised "American Standard Building Code Requirements (A58-1.1955)".

The maximum probable wind velocity to which the paper refers should then include a gust factor in order to adjust the wind pressure in the ratio of the square of the design wind velocity. Furthermore, the increase of wind pressure with altitude, including the gust effect, may amount to +20% and +35% for elevations about ground of 100' and 200' respectively.(4)

The shape factors as defined previously are in fairly good agreement with

test results effected over the last 25 years in various countries and cover the increase due to a possible variation of pitch angle.

Since these coefficients are determined for a static wind pressure, a special study should be made concerning the dynamic effect due to the wind on slender structures. It should also be noted that the shape factor for trusses may be increased appreciably for a very low solidity ratio as most of the tests were conducted on models of trusse-bridges with a solidity ratio higher than 0.20. The shape factor for a single trusse varies for instance from 1.60 to 2.00 for a solidity ratio varying from 0.20 to 0.[4]

The shape factor for a girder bridge does not vary substantially with the number of girders, the whole bridge performing almost like a closed box.[3] Yet, the shape factor for trussed-bridges is much more affected by the number of trusses and a special provision should be made concerning multi-trusse-bridges with appropriate shielding coefficients.

The variation of lateral wind force as a function of yaw is also in very good agreement with test results and foreign codes.[2] The longitudinal wind force is somewhat conservative according to some recent tests,[3] but the margin will be reduced for the case of prestressed concrete bridges with deep full-web diaphragms.

On a more general basis it should be mentioned that drag coefficients may, for round numbers, be reduced by 30% or 60%, depending on the value of the Reynolds number.[4]

Since the "Criteria for Prestressed Concrete Bridges" cover mainly the problems arising from the use of prestressed concrete as a material, the AASHO Specifications have to be used for such problems as the distribution of live loads. The new revisions are mentioning a distribution factor equal to S/5.5 for a span: S, not over 14 feet, and it seems very questionable as how much should be this factor for prestressed concrete bridges. The BPR criteria do not specify any spacing or particular type of diaphragm and various possibilities of interaction between the beams will be covered by a single coefficient.

It is furthermore of the writer's opinion that the Specifications should give the opportunity to design a bridge as a grid floor with diaphragms used as part of the structural system. Such type of design has already been applied successfully in Europe and Canada[5] for concrete and steel bridges.

The revision of the criteria governing intermediate stiffeners is very timely in view of the recent improvements made on the theoretical basis of the problem. However, the need of "Transverse stiffeners made in pairs and fitted at both ends to the flanges" does not seem to apply for a composite section for which the danger of warping is considerably reduced.

The reduction of the design shrinkage coefficient from .00004 to .00002 will be welcome since the use of the former value added to the conventional termal forces was most of the time leading to very unrealistic design, particularly columns and footings.

The study of the expansive characteristics of concrete is still at its beginning and, whether natural or artificially provoked, concrete expansion still remains an indeterminate phenomena. The coefficients of thermal expansion for concrete and steel may vary for some aggregates in the ratio one to two. Besides, high-alkali cements may cause a long-term growth detrimental to the composite section. It has thus been advised[6] to use, particularly for the concrete of composite structures, low-alkali cement and chemically inert aggregate.

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3. "Investigation of Wind Forces and Highway Bridges," Highway Research Board, Special Reports 10, 1953.

4. Cohen, Ed. and Perrin, H., "Design of Multilevel Guyed Towers: Wind Loading," Proceeding ASCE No. 1355, 1957.

5. Lount, A. M., "Distribution of Loads on Bridge Decks," Proceedings ASCE No. 1303, 1957.

6. "Composite Construction of Bridges using Steel and Concrete." Paper presented by R. David and G. Meierhof before the annual meeting of the Engineering Institution of Canada, June 1957.

ROBERT DAVID.[1]—This paper is of great importance to bridge engineers, especially the section covering composite beam design. Many engineers have felt reluctant to use composite sections in the past because of a lack of a suitable specification covering their use. It is hoped that this reluctance will be dispelled with the inclusion of a more comprehensive section on composite design in the new AASHO Specification so that full advantage may be taken of the economy of this type of construction. For this work the authors are to be complimented.

There are, however, some points regarding the interaction of steel and concrete that appear worthy of serious consideration which should be brought to the attention of the authors.

1) On page 1320-4 it is stated: "The coefficient of linear expansion of steel is 0.0000065 per degree F. and for concrete it is 0.000006."

Results of many tests on concrete have shown that the coefficient of thermal expansion or contraction varies from 2 to 9×10^{-6} per degree F., dependent to a large extent on the aggregates used. Calcareous aggregates, for instance, have a low thermal coefficient and in the Appendix to this discussion it is shown that the intensity of thermal stresses induced in steel and concrete in intimate contact are quite large when the concrete has a low thermal coefficient and the range of temperature variation is large.

Last June Dr. G. Meyerhof and myself presented a paper on composite construction at the Engineering Institute of Canada General Convention. In this paper equations were developed which permit the computation of thermal stresses. Similar equations had also been developed in Germany for computing stresses due to a difference in temperature between the concrete slab and the steel beam.

It is of interest to note that these equations can be used for ordinary reinforced concrete sections also. In this case the longitudinal forces developed along the reinforcing rods are not negligible and may be a contributing factor in causing spalling, especially where such stresses combine with shear stresses.

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1. District Engineer, Canadian Inst. of Steel Const., Montreal, Canada.

2) Page 1320-32 - Composite beams - Proposed specifications. "If Concrete with expansive characteristics should be used, composite design should be used with caution, and provision must be made in the design to accommodate the expansion."

This very sound advice should be given for any type of construction, and it would be even wiser to say that expansive concretes should be first detected and never used in any circumstances. Numerous studies during the last decade have brought to light a number of aggregates which should not be used for concrete.

One paper presented by Messrs. Swenson and Chaly - (A.C.I., May, 1956 - pages 52-58) sums up their own work and other papers. It shows that concrete expansion is due to physical and chemical characteristics. Deleterious aggregates are classified. This important work enables the engineer to eliminate concretes which may be harmful in the long run.

3) Page 1320-52 - "The effect of creep shall be considered in the design of composite beams which have dead load acting on the composite section. In such structures, stresses and horizontal shears produced by dead loads acting on the composite section shall be computed for "n", as given in article 3-4-11 or for this value multiplied by 3, whichever gives the higher stresses and shears".

Most laboratory tests concerning shrinkage and creep have been made on samples which do not include reinforcing bars and it is thus difficult to know exactly the total expansion or contraction which can be expected on a job after a few years and whether such expansion or contraction shows a tendency to stop, which is not always the case. From some experiments it has been deducted that shrinkage and creep are less important in the case of reinforced concrete and coefficients have been reduced from 4 to 2×10^{-4} which is perhaps still a high figure, since in the case of bridges, concreting is done by sections and thus normally this figure might be reduced to 1×10^{-4}.

Exact computation by the proposed method shows that the increase of the stresses in the lower fiber depends largely on the position of the steel center of gravity and when cover plates are used, tension is usually increased only by about 0.5 kips per square inch.

CONCLUSION

"Mathematical analysis seems frequently to be regarded as the essence of theory. Assumptions with equal frequency are advanced with little or no discussion or logical justification. I definitely regard the assumptions underlying any theory as the most basic and important elements of a theory."

This quotation of Professor Van den Broek's evaluation of Plastic Design is certainly most appropriate in this discussion.

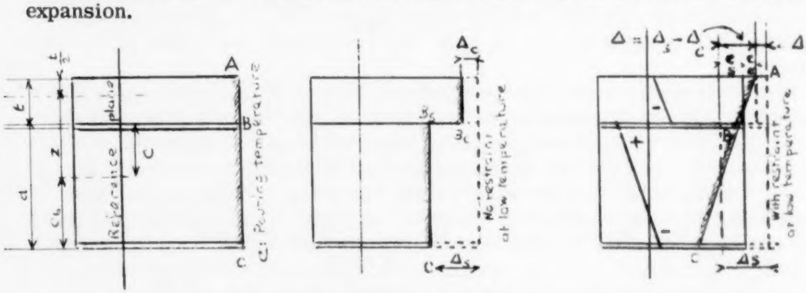
From the study of numerous test reports one reaches the conclusion that too often in design there was no consideration as to the possibility that concrete and steel may behave differently. Steel structures supporting a concrete slab are still computed on the assumption that concrete is just a dead load whereas numerous tests show that they should be considered composite. Such slabs, due to projections of rivets, cover plates, beam flanges, friction, and bond etc. under live loads, including impacts, have the same strains as the steel surfaces which they contact. Thus one cannot understand why designers take so much pain to make long mathematical analyses which are inaccurate, since they ignore proven facts.

On most structures, steel and concrete are closely associated, (e.g., steel bridges with concrete slabs, composite, reinforced or prestressed concrete bridges). Thus, a perfect knowledge of the properties of the materials to be used is necessary. If the designer, having this knowledge, is governed thereby, then structures will be safe and last for many years.

APPENDIX

Computation of Stresses in a Composite Section due to a Difference Between the Thermal Coefficients of Expansion and Contraction of Concrete and Steel

The following diagrams illustrate the strains that are produced due to a difference in the rate of thermal contraction of concrete and steel used as composite section (assuming that the steel thermal coefficient is larger than the concrete thermal coefficient) or due to the concrete chemical or physical expansion.



In the composite section strains are set up due to the restraint at the contact surface.

Let Δ_s = steel contraction due to temperature change assuming no restraint

Δc = concrete contraction due to temperature change assuming no restraint

Δ = $\Delta_s - \Delta_c$

θ s = steel strain due to composite action at the contact plane

θ c = concrete strain due to composite action at the contact plane

A_s, I_s, M_s = steel area, moment of inertia and moment

A_c, I_c, M_c = concrete area, moment of inertia and moment

N_c = longitudinal shear in concrete at the contact surface

N_s = longitudinal shear in steel at the contact surface

E_s = Modulus of elasticity for steel

E_c = Modulus of elasticity for concrete

n = $\dfrac{E_s}{E_c}$ = modular ratio

d = depth of steel beam

t = slab thickness

z = distance between C.g. of slab and C.g. of steel beam = t/2 + C

c = distance between top flange of steel beam and C.g. of steel beam.

Since the beam is in equilibrium

$$N_s = N_c = N$$

Since there is no slip on the contact surface

$$\theta_s + \theta_c = \Delta \qquad (1)$$

$$\frac{f_s}{E_s} + \frac{f_c}{E_c} = \Delta \qquad (2)$$

The strain diagram can be broken down into two components namely a diagram representing strains due to pure tension or compression plus a diagram representing strains due to bending stresses caused by the eccentricity of the shear forces.

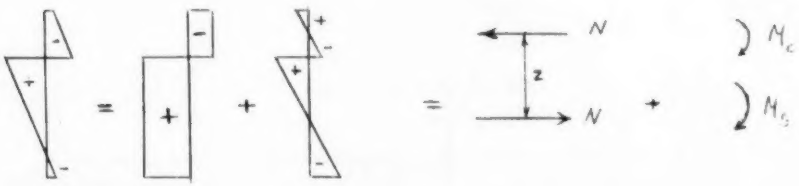
Since the beam is composite the slab and the steel beam must have the same radius of curvature P

$$P = \frac{M}{EI} = \frac{M_s}{E_s I_s} = \frac{M_c}{E_c I_c} = \frac{N.z}{E_s I_s + E_c I_c} \qquad (3)$$

Since all the forces are in equilibrium

$$N.z. = M_c + M_s \qquad (4)$$

If all the stresses are within the elastic range the stress diagram will be identical in appearance to the strain diagram



$$f_{st} = \frac{N}{A_s} + \frac{M_s}{I_s}.c \text{ - for a beam with no cover plate } C = d/2 \qquad (5)$$

$$f_{cb} = \frac{N}{A_s} + \frac{M_c.t}{I_c.2} \qquad (6)$$

Now

$$\frac{Mc}{I_c} = \frac{N.z}{n.I_s + I_c} \quad \text{from Eq. (3)} \qquad (7)$$

Letting

$$\frac{1}{nI_s + I_c} = \alpha$$

$$\frac{M_c}{I_c} = N.z.\ \alpha \tag{8}$$

$$\frac{M_s}{I_s} = N.z.n.\ \alpha \tag{9}$$

Substituting in Eq. we obtain

$$N = \frac{\Delta\ E_s}{\frac{1}{A_s} + \frac{n}{A_c} + n.z^2.\alpha} \tag{10}$$

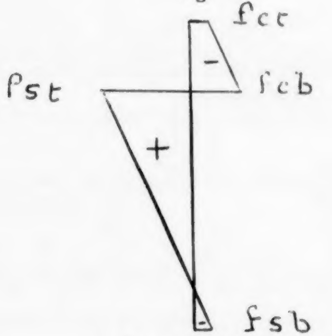
Whence

$$f_{cb} = \frac{N}{A_c} + N.z.\alpha.t/2 \tag{11}$$

$$f_{ct} = \frac{N}{A_c} - N.z.\alpha.t/2 \tag{12}$$

$$f_{st} = \frac{N}{A_s} + N.z.n.\alpha.c \tag{13}$$

$$f_{sb} = \frac{N}{A_s} - N.z.n.\alpha.c_b \tag{14}$$



Example 1 (Fig. 1)

Slab 84" x 7.25" $A_c = 609\ in^2$

$I_c = 2670\ in^4$

Steel beam 24 WF 76 + C.P. 12" x 11/16"

$A_s = 30.57\ in^2$

$\bar{y} = 9.35"$

$I_s = 3000\ in^4$

Assuming $n = 10$ $E_s = 30 \times 10^6$

$\alpha = 3.06 \times 10^{-5}$ $a = 18.88$ in.

$N = 1.9 \times 10^8 . \Delta$ lbs.

$$f_{st} = +22.72 \times 10^6 \Delta$$

$$f_{sb} = -\ 4.23 \times 10^6 \Delta$$

$$f_{cb} = -\ 7.1 \times 10^5 \Delta$$

$$f_{ct} = +\ 0.86 \times 10^5 \Delta$$

Supposing we had a thermal coefficient for concrete: $4 \times 10^{-6}/^\circ$ F. and that the setting temp. had been 80° F.

The stresses occurring at -30° F (a 110° drop) would be as follows:

$$\Delta = 110 \times (6.5-4) \times 10^{-6} = 2.75 \times 10^{-4}$$

$$f_{st} = 22.72 \times 10^6 \times 2.75 \times 10^{-4} = +6250 \text{ psi (tens.)}$$

$$f_{sb} = -4.23 \times 10^6 \times 2.75 \times 10^{-4} = -1165 \text{ psi (comp.)}$$

$$f_{cb} = -7.1 \times 10^5 \times 2.75 \times 10^{-4} = -\ 195 \text{ psi (comp.)}$$

$$f_{ct} = +0.86 \times 10^5 \times 2.75 \times 10^{-4} = +\ 24 \text{ psi (tens.)}$$

$$N = 52.300 \text{ lbs.}$$

N is the longitudinal shear stress all along the beam between the contact surfaces of the slab and the top flange of the steel beam.

It is of interest to note that the stresses due to possible concrete growth may be calculated by means of the same equations as developed for differences in thermal contraction, as the effects are exactly the same.

For instance if we had a concrete expansion over a period of time equivalent to 3×10^{-4} in/in. in an unrestrained slab the stress would be

$$f_{st} = +6820 \text{ psi (tens.)}$$

$$f_{sb} = -1270 \text{ psi (comp.)}$$

$$f_{cb} = -\ 213 \text{ psi (comp.)}$$

$$f_{ct} = +\ 26 \text{ psi (tens.)}$$

If we had a concrete with a thermal contraction coefficient of $4 \times 10^{-6}/^\circ$ F. which also exhibited expansive growth over a period of time of 3×10^{-4} in/in., we find that at low temperatures the stresses caused by the two conditions are directly additive. Thus at -30° F. the stresses would be

$$f_{st} = +13070 \text{ psi (tens.)}$$

$$f_{sb} = -\ 2430 \text{ psi (comp.)}$$

$$f_{cb} = -\ 408 \text{ psi (comp.)}$$

$$f_{ct} = +\ 50 \text{ psi (tens.)}$$

Note that in the case of simple span composite sections the above stresses except for fc, are opposite to those caused by loading conditions. However in continuous spans the steel stresses in particular are directly additive to the negative moment stresses over the supports. The shear stresses are usually highest at this section also.

Example 2 (Fig. 2)

Slab: $A_c = 864\ in^2$ $I_c = 4605\ in^4$ $z = 49.82"$

Steel: $I_s = 109.800\ in^4$ $A_s = 102.4\ in^2$

$\alpha = 9.08 \times 10^{-7}$ $N = 6.85 \times 10^8 \times \Delta$

Under the same conditions as the first example:

$\Delta = 2.75 \times 10^4$ N = 188.500 lbs.

$f_{st} = +5750$ psi (tens.) $f_{cb} = -252$ psi (comp.)

$f_{sb} = -\ 860$ psi (comp.) $f_{ct} = -184$ psi (comp.)

2nd assumption as in previous example:

$f_{st} = 12000$ psi $f_{cb} = -520$ psi

$f_{sb} = -1800$ psi $f_{ct} = -385$ psi

2nd assumption as in previous example
$\Delta = 3 \times 10^{-4}$ (concrete expansion)

$f_{st} = +\ 6270$ psi (tension)

$f_{sb} = -\ 940$ psi (comp.)

$f_{cb} = -\ 275$ psi (comp.)

$f_{ct} = -\ 200$ psi (comp.)

3rd assumption as in previous example (thermal difference and concrete expansion)

$f_{st} = +12000$ psi (tension)

$f_{sb} = -\ 1800$ psi (comp.)

$f_{cb} = -\ 530$ psi (comp.)

$f_{ct} = -\ 385$ psi (comp.)

Example 3 (Fig. 3)

Slab: 72" x 6-1/2"
Beam: 33-1/2" x 12"

$A_s = 7.2\ in^2$ $n = 10$ $d = 23.5"$

$I_c = 132.550\ in^4$ $I_s = 0$

$$N_s = N_c = N$$

$$M_c = N.z$$

$$C_s + C_c = \Delta \quad \text{at the plane of contact}$$

$$C_s = \frac{N}{A_sE_s} \qquad C_c = \frac{N}{A_cE_c} + \frac{N.z^2}{E_cI_c}$$

$$\frac{N}{A_sE_s} + \frac{N}{A_cE_c} + \frac{N.z^2}{E_cI_c} = \Delta$$

$$\frac{N}{30 \times 10^6} \int \frac{1}{7.2} + \frac{1}{87} + \frac{23.5^2}{13255} \int = \Delta$$

$$N\,(.139 + .0115 + .0416) = 30 \times 10^6 \Delta$$

$$.1921\ N = 30 \times 10^6 \Delta$$

$$N = 15.6 \times 10^7 \Delta$$

If we consider the first assumption as in example 1 we have

$$\Delta = 2.75 \times 10^{-4}$$

$$N = 15.6 \times 2.75 \times 10^3 = 43000 \text{ lbs.}$$

$$f_s = \frac{43000}{7.2} = 6000 \text{ psi (tension)} = 6000 \text{ psi (tension)}$$

$$f_{cb} = \frac{-43000}{870} - \frac{43000 \times 23.5 \times 27.5}{132\ 550} = 260 \text{ psi (compression)}$$

$$f_{ct} = -50 + \frac{43000 \times 23.5 \times 14.5}{132\ 550} = 60 \text{ psi (tension)}$$

Considering the 2nd assumption as in previous examples

$\Delta = 3 \times 10^{-4}$ (concrete expansion)

$f_s = 6500$ psi (tension)

$f_{cb} = 280$ psi (comp.)

$f_{ct} = 65$ psi (tension)

And with the 3rd assumption as in previous examples

$f_s = 12500$ psi (tension)

$f_{cb} = 540$ psi (comp.)

$f_{ct} = 125$ psi (tension)

N.B. A 3×10^{-4} inch per inch concrete expansion is a very conservative figure and in some cases it was found that it reached 20 to 30×10^{-4}. Thus stresses calculated above are increased up to the steel elastic limit.

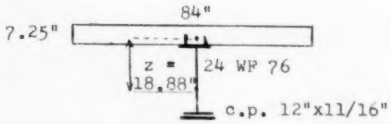


FIG. I
EXAMPLE 1

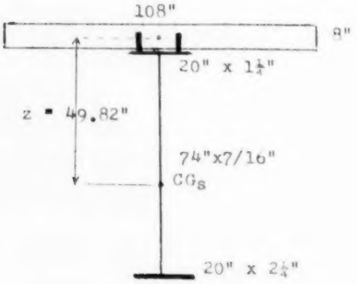


FIG. II
EXAMPLE 2

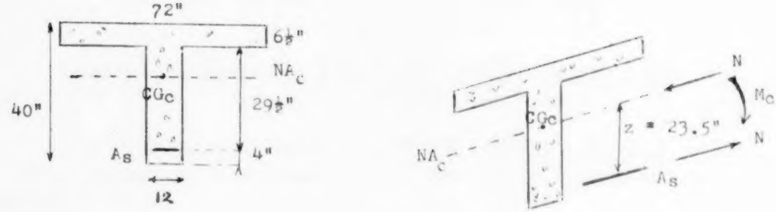


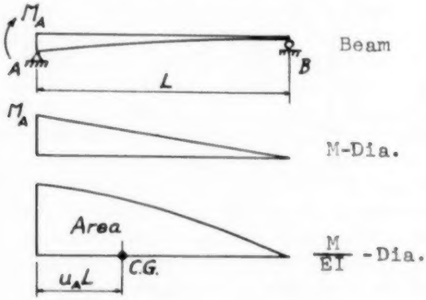
FIG. III
EXAMPLE 3
(REINFORCED CONCRETE BEAM)

PIN-ENDED GABLE FRAMES[a]

Discussion by Kurt H. Gerstle

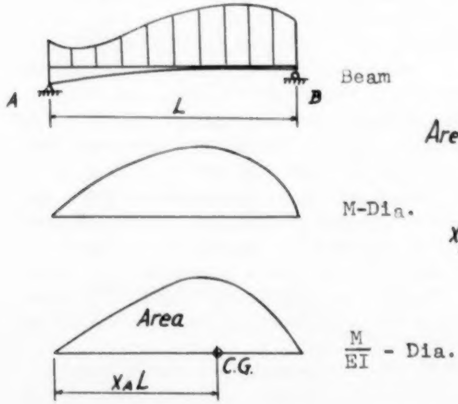
KURT H. GERSTLE,[1] J.M. ASCE.—Prof. Chinn's ingenious method for utilizing tabulated moment-distribution constants in consistent deformation analysis will be welcomed by many analysts. A somewhat different version of the same process has been presented for many years by Prof. Bruce Jameyson of the University of California at Berkeley. Whereas Prof. Chinn expresses deformations in terms of angle change between chords, this latter method gives the deformations of the frame in terms of the conventional moment areas or elastic weights of members.

The following expressions for converting moment diagrams of non-prismatic members to the corresponding elastic weights have been extracted from mimeographed notes of Prof. Jameyson's.



$$Area = \frac{M_{AB}}{K_{AB}}\left(\frac{1+C_{BA}}{1-C_{AB}C_{BA}}\right);$$

$$u_A = \frac{C_{BA}}{1+C_{BA}};$$



$$Area = -\left[\frac{M^F_{AB}}{K_{AB}}\left(\frac{1+C_{BA}}{1-C_{AB}C_{BA}}\right) + \frac{M^F_{BA}}{K_{BA}}\left(\frac{1+C_{AB}}{1-C_{AB}C_{BA}}\right)\right];$$

$$x_A = \frac{M^F_{BA} + M^F_{AB}C_{AB}}{M^F_{BA}(1+C_{AB}) + M^F_{AB}\left(\frac{C_{AB}}{C_{BA}} + C_{AB}\right)};$$

a. Proc. Paper 1353, September 1957, by James Chinn.

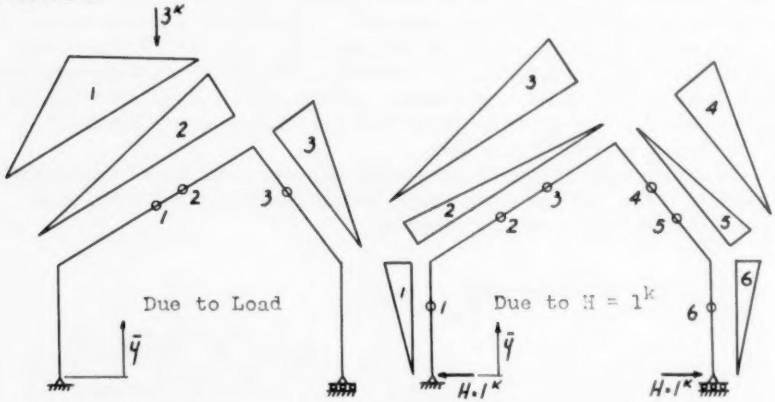
1. Instructor, Dept. of Civ. Eng., Univ. of Colorado, Boulder, Col.

The symbols in these formulas have the same meaning as in Prof. Chinn's paper. These expressions may be derived by slope deflection and moment area principles.

In applying these formulas to the analysis of a structure, the statically determinate moment diagram is plotted by parts, and the corresponding values and locations of the elastic weights are computed. With the elastic weights indicated on the frame, deformations may be obtained by conventional methods.

The method is shown applied to the author's first example problem. Though of roughly the same amount of work, it is possible that this procedure might be of more appeal to the analyst familiar with deflection calculations by moment area.

For statement of problem and properties of members, see Example Problem 1.



Part	$\frac{M}{EI}$ Area	u	uL	$\bar{y}$	Area. $\bar{y}$
Due to Load					
1	16.85	.517	18.60'	27.85'	470
2	10.50	.375	13.50'	29.95'	314
3	8.29	.371	8.90'	30.03'	249
					Δ Load = 1033
Due to H = 1k					
1	4.35	.415	7.47'	10.53'	46
2	11.82	.375	13.50'	25.15'	298
3	24.40	.375	13.50'	29.95'	730
4	19.25	.371	8.90'	30.03'	579
5	9.35	.371	8.90'	25.07'	235
6	6.00	.398	7.15'	10.85'	65
					Δ H = 1 = 1953

$$\therefore\ H = \frac{-1033}{1953} = \underline{-.53^k}$$

UNIVERSITY RESEARCH IN STRUCTURAL ENGINEERING[a]

Discussion by E. Neil W. Lane

E. NEIL W. LANE,[1] A.M. ASCE.—The history of the most successful research, including that for structural engineering, is the story of men of peculiar mien, in some respects, and special circumstances (most often) occurring at the same time and place; perhaps, somewhat as a few amoebas got together down through the ages to produce man.

The most unfortunate result of the process of natural selection is the laissez faire inefficiency with which the economic and social process serves to provide the comparatively few round-peg men of research aptitude with the square-hole jobs for the family's bread and butter, as well as that sometimes overdone fetish known as experience. Growing experience certainly is not decried and it is realized that much of life must be routine. Without it the unusual would not provide the incentive thrill of being outstanding when the results are favorable. At the other end of the scale, one finds the need, money, and materials in various combinations, but without the particular mental catalyst to produce research or develop fruit.

As seen this way it becomes very similar to the problem of peak load expectancy when the theory of probabilities is applied to a stream of bridge traffic if the professional research pinnacles are likened to a 'mental' truck. As in the intentional grouping of the trucks of one concern in a fleet; likewise, one has the intentional grouping of outstanding research men at universities and specialized organizations. And, at the most improbable expectancy for a haphazard peak load, one finds the counterpart in the 'once-in-a-lifetime' 'out-of-the-clear-sky' type of engineer who makes a name for himself. Often his one idea or result is more important than several of the others put together as far as impact on the profession is concerned. Undoubtedly, among the 500,000 working engineers and scientists in the U. S. A., the theories of probability and distribution hold well.

As to the relative importance of the ingredients of the research potion, one in the writer's position of professional prejudice has little trouble in agreeing that the active capable mind is tops. Where freedom of thought, and a chance to apply this with proper reward, is provided, the results are usually gratifying in a reasonable time. In this connection it must be remembered that new thoughts do not occur every day, so to say, and only sometimes, under pressure. As DuPont has said, "Brains is where you find it" and like the package, it comes in many varied forms—some hardly recognizable until the product evolves. Admittedly the world's most scarce article and most valuable gem, it should be guided (or led to roam) in its field with the greatest of

a. Proc. Paper 1357, September, 1957, by Frank Baron.

1. Structural Engr., Lodi, Calif.

efficiency for reasonable production (not burnt out with pressure of routine commercial headaches). Ample reward for its achievements will solve the problem of inducing young men to become teachers, engineers, and scientists. Too much advantage has been taken to the detriment of the technical expert by organizational and promotional groups in times past and it is believed that the long term pendulum must swing the other way for quite a few years.

The general public is still too well ingrained with the prejudicial preconceptions of the eggheaded impractical professor at the opposite end of the engineering line from the open-shirted, result-producing S.I. pictured dismounting from a tractor on our cigarette advertisements. While the profession is partly at fault for this, in large part it is not; and, indirectly or otherwise, the public attitude is in large part responsible for the fact that educational funds, particularly for research, are among the first appropriations to be cut when appearances forebear an impending business decline. This often costs the expenditures to date on previously incompleted work, if and when the loose ends can be picked up after the lapse of several years. Usually the times and personnel are so different by then that the value of the previous research is largely lost. This of course applies to the individual project type of investigation. Long-range studies of more than twenty year periods suffer least because the usual drop out period of two to five year appropriations can sometimes be interpolated from subsequent data. Continuous or cyclical work over a period of less than twenty years is often ruined because the period of lapse is usually too large a portion of the total. In nearly every instance, it can be shown that the cost of the research was more than compensated for by the reduced cost of the construction using the data or else a more profitable service life of the structure was provided. No research at all should be indefensible situation for a university but is at times beyond its control. Cuts in research work can only be seen as robbing Peter to pay Paul, an expensive snare and a delusion.

Instinctively the laity looks to its universities as a sort of overgrown animated encyclopedia, always available for a friendly consultation or a few pamphlets when a problem arises above the ordinary level. Nowhere is this developed to a better extent than the liaison and pleasant support offered by the farmer to his agricultural college. In this respect the engineering situation is usually good but can be bettered at times with corresponding understanding and assistance with appropriations at the right times.

Another matter should be plugged for the defense of some of our very best teachers, and by that is meant those that did the most of real value for their students as recognized in later years. Particularly in the field of basic courses, adeptness in the field of research serves only to keep the professor alive and interested. There is nothing in calculus that helps the grade school teach arithmetic to Johnny. In the respect that the author is primarily concerned with senior and graduate grade work, the writer is quite in agreement with his views with the exception of the extent to which they are indicated.

While logic and the processes of inductive and deductive reasoning, once popular as a course, are now out of date somewhat and planted in an incidental manner in other courses as the scientific process, no amount of the instilling of such principles is as effective as the nearly instinctive use of them by the searcher with the energy and alertness of interest applied to his work and with the incentives of adequate remuneration and recognition for work well done. Such a teacher attracts interest in his students and while often diverting from the main theme of the course, nonetheless gains many compensating advantages in himself and his students.

Again it must be emphasized that the development of structural engineering (as all other) research is the history of unusual and outstanding people of unique experience. In the smaller schools, or those less well endowed or financed, one often finds the field completely overshadowed by the outstanding ability and reputation of one or a few men who really are the school and all that is in it. But, with the larger schools where there is a full complement of top men in their respective subjects there arises a new need. That is the managerial or executive type of civil engineer (in this instance) with broad technical training to assure a balanced program of research and its understanding administration over the structural, hydraulic, transportation and other subdivisions. This arises from the belief that others sometimes see us in better perspective in our surroundings than one can hope to visualize himself however hard one may try.

Also along another line, great tribute is overdue to many of the publicly anonymous individuals that worked on secret military research work during and after the war for governmental agencies and private corporations. Many of the university's personnel came under this category and performed heroically in any way real value can be measured.

Another group due thanks galore is the great army of helpers, assistants and aids that do the chore work for the busy investigator making it possible for him to multiply his output severalfold.

There is much to be said regarding the granting of patents to university personnel, but it is believed to be very much within their rights to have university assistance in these matters as a result of a professor's research and development much as the U. S. Army or Navy aids its employees. There should be specific contract restraints where a private party wishes to retain patent rights on work done by and at a university. There is much more opportunity for chemical or mechanical engineers than for structurals in this respect.

With the current excess of demand for a Ph.D. it is hoped that the universities will bend every resistance to a change of requirements diluting the quality of training and undermining those that have already put forth the time, money, and effort for its accomplishment. There has been far too much degradation of the school structure to date. Let us not betray those dependent on us and the faithful. Whether intentional or not, the result is the same, the sabotage of educational quality to satisfy pressure groups has only one end and that is communist in nature. The time and means of obtaining more and better research engineers is seven years earlier and a fair incentive, mostly financial. A lot can be said in favor of and little against a two-year pre-engineering (or B.A.) prerequisite for the engineering colleges. The writer is highly in favor of a two year head start for most universities at this time.

The writer would like to suggest the EJC, or better perhaps the ECPD, as a national depository for subjects to be investigated as structural engineering research much as the Defense Department has a list of needed projects for inventors. From the list each school could take a study of its own choosing, as facilities become available, or it could add its own developments as a check against duplication of effort as requests arrive locally from commercial agencies. This depository would make it easier to start with what is known, keep loose ends continually working toward a planned goal, be a source of aid for obtaining best facilities for the job request and could assist in expediting this kind of work in war times as well as peace. The ASCE and ASME membership would be the two professional groups cooperating for the most part for

the stimulation and production of much needed structural engineering research and this could be obtained with comparative ease and much mutual satisfaction it is believed.

Outside of ample cash reserves and efficient procedures at this phase of the business cycle, there is no better insurance for staying in a competitive business than for management to spend judicious sums for research leading to cost reductions for time, materials and money. Structural engineering research (whether it be for a better machine or girder, better processes of assembly, or better materials) is no exception. The total amounts involved in its industry is very large for the final products, but the cost of the competitive insurance is very low, especially when expressed as a percentage.

The author has written a very unusual article for this journal and it is fine to see.

WOOD DIAPHRAGMS[a]

Discussion by E. George Stern

E. GEORGE STERN,[1] M. ASCE.—It is stated that (1) "the properties of solid diaphragms are largely dependent on the nailing of the laminations", (2) "the strength and rigidity (of diagonally sheathed lumber diaphragms) is limited principally by the nailing pattern . . .", (3) "the nailing schedule for the plywood (sheathed diaphragm) . . . generally determines allowable shears". In light of this, it is surprising how relatively few research efforts have been put into improved fastening of diaphragm elements.

To shed some light onto possibly better fastening procedures for the components of wood diaphragms, studies of nails specially designed for fastening plywood diaphragm sheathing were recently initiated in the Wood Research Laboratory of Virginia Polytechnic Institute.

In light of the fact that the nails have to be closely spaced, that is, 3 to 6 inches on centers for fastening the diaphragm sheathing along the plywood panel perimeters, many nails are to be driven. Hence, the use of short nails can be advantageous. Such short nails are feasible and provide the required holding power to prevent panel buckling if they are properly threaded. Because of their limited length, the short threaded nails can cost less, are less bulky, thus easier to handle, and are driven faster, hence, can reduce the overall cost of construction.

To transmit large shear loads, these nails are provided with extra large shank diameters. A comparison of the dimensions of the old and the recently introduced nails is appropriate:

Common Wire Nails		Threaded Diaphragm Nails	
Size	Count/Pound	Size	Count/Pound
6d—2" x 0.113"	167	1-1/2" x 0.135"	160
8d—2-1/2" x 0.131"	101	1-3/4" x 0.135"	140
10d—3" x 0.162"	66	2" x 0.148"	100

It is evident that these threaded nails can go considerably further than the longer common wire nails. The effectiveness of these so-called "Hi-Load" nails is shown in V.P.I. Wood Research Laboratory Bulletin No. 32 (in print). The principal conclusions, as presented in this Bulletin, are given below:

(1) In comparison with the holding power of the 2-1/2" common wire nail, the one-third lighter 1-1/2" and same-weight 2" threaded nails

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a. Proc. Paper 1433, November, 1957.

1. E. B. Norris Research Professor of Wood Construction, Virginia Polytechnic Inst., Blacksburg, Va.

may be three and five times, respectively, more effective and five times more efficient under given conditions.

(2) The shorter threaded nails can transmit the same and, under given conditions, a slightly larger diaphragm shear load than the 2-1/2" common wire nail. Yet, the efficiency, on a uniform weight basis, of the 1-1/2" threaded nail may be twice that of the 2-1/2" common wire nail.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper numbers are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1449 is identified as 1449 (HY 6) which indicates that the paper is contained in the sixth issue of the Journal of the Hydraulics Division during 1957.

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JANUARY: 1136(CP1), 1137(CP1), 1138(EM1), 1139(EM1), 1140(EM1), 1141(EM1), 1142(SM1), 1143(SM1), 1144(SM1), 1145(SM1), 1146(ST1), 1147(ST1), 1148(ST1), 1149(ST1), 1150(ST1), 1151(ST1), 1152(CP1)[c], 1153(HW1), 1154(EM1)[c], 1155(SM1)[c], 1156(ST1)[c], 1157(EM1), 1158 (EM1), 1159(SM1), 1160(SM1), 1161(SM1).

FEBRUARY: 1162(HY1), 1163(HY1), 1164(HY1), 1165(HY1), 1166(HY1), 1167(HY1), 1168(SA1), 1169(SA1), 1170(SA1), 1171(SA1), 1172(SA1), 1173(SA1), 1174(SA1), 1175(SA1), 1176(SA1), 1177(HY1)[c], 1178(SA1), 1179(SA1), 1180(SA1), 1181(SA1), 1182(PO1), 1183(PO1), 1184(PO1), 1185(PO1)[c].

MARCH: 1186(ST2), 1187(ST2), 1188(ST2), 1189(ST2), 1190(ST2), 1191(ST2), 1192(ST2)c, 1193 (PL1), 1194(PL1), 1195(PL1).

APRIL: 1196(EM2), 1197(HY2), 1198(HY2), 1199(HY2), 1200(HY2), 1201(HY2), 1202(HY2), 1203 (SA2), 1204(SM2), 1205(SM2), 1206(SM2), 1207(SM2), 1208(WW1), 1209(WW1), 1210(WW1), 1211(WW1), 1212(EM2), 1213(EM2), 1214(EM2), 1215(PO2), 1216(PO2), 1217(PO2), 1218 (SA2), 1219(SA2), 1220(SA2), 1221(SA2), 1222(SA2), 1223(SA2), 1224(SA2), 1225(PO)[c], 1226 (WW1)[c], 1227(SA2)[c], 1228(SM2)[c], 1229(EM2)[c], 1230(HY2)[c].

MAY: 1231(ST3), 1232(ST3), 1233(ST3), 1234(ST3), 1235(IR1), 1236(IR1), 1237(WW2), 1238 (WW2), 1239(WW2), 1240(WW2), 1241(WW2), 1242(WW2), 1243(WW2), 1244(HW2), 1245(HW2), 1246(HW2), 1247(HW2), 1248(WW2), 1249(HW2), 1250(HW2), 1251(WW2), 1252(WW2), 1253 (IR1), 1254(ST3), 1255(ST3), 1256(HW2), 1257(IR1)[c], 1258(HW2)[c], 1259(ST3)[c].

JUNE: 1260(HY3), 1261(HY3), 1262(HY3), 1263(HY3), 1264(HY3), 1265(HY3), 1266(HY3), 1267 (PO3), 1268(PO3), 1269(SA3), 1270(SA3), 1271(SA3), 1272(SA3), 1273(SA3), 1274(SA3), 1275 (SA3), 1276(SA3), 1277(HY3), 1278(HY3), 1279(PL2), 1280(PL2), 1281(PL2), 1282(SA3), 1283 (HY3)[c], 1284(PO3), 1285(PO3), 1286(PO3), 1287(PO3)[c], 1288(SA3)[c].

JULY: 1289(SM3), 1290(EM3), 1291(EM3), 1292(EM3), 1293(EM3), 1294(HW3), 1295(HW3), 1296 (HW3), 1297(HW3), 1298(HW3), 1299(SM3), 1300(SM3), 1301(SM3), 1302(ST4), 1303(ST4), 1304 (ST4), 1305(SU1), 1306(SU1), 1307(SU1), 1308(ST4), 1309(SM3), 1310(SU1)[c], 1311(EM3)[c], 1312 (ST4), 1313(ST4), 1314(ST4), 1315(ST4), 1316(ST4), 1317(ST4), 1318(ST4), 1319(SM3)[c], 1320 (ST4), 1321(ST4), 1322(EM3), 1323(AT1), 1324(AT1), 1325(AT1), 1326(AT1), 1327(AT1), 1328 (AT1)[c], 1329(ST4)[c].

AUGUST: 1330(HY4), 1331(HY4), 1332(HY4), 1333(SA4), 1334(SA4), 1335(SA4), 1336(SA4), 1337 (SA4), 1338(SA4), 1339(CO1), 1340(CO1), 1341(CO1), 1342(CO1), 1343(CO1), 1344(PO4), 1345 (HY4), 1346(PO4)[c], 1347(BD1), 1348(HY4)[c], 1349(SA4)[c], 1350(PO4), 1351(PO4).

SEPTEMBER: 1352(IR2), 1353(ST5), 1354(ST5), 1355(ST5), 1356(ST5), 1357(ST5), 1358(ST5), 1359(IR2), 1360(IR2), 1361(ST5), 1362(IR2), 1363(IR2), 1364(IR2), 1365(WW3), 1366(WW3), 1367(WW3), 1368(WW3), 1369(WW3), 1370(WW3), 1371(HW4), 1372(HW4), 1373(HW4), 1374 (HW4), 1375(PL3), 1376(PL3), 1377(IR2)[c], 1378(HW4)[c], 1379(IR2), 1380(HW4), 1381(WW3)[c], 1382(ST5)[c], 1383(PL3)[c], 1384(IR2), 1385(HW4), 1386(HW4).

OCTOBER: 1387(CP2), 1388(CP2), 1389(EM4), 1390(EM4), 1391(HY5), 1392(HY5), 1393(HY5), 1394(HY5), 1395(HY5), 1396(PO5), 1397(PO5), 1398(PO5), 1399(EM4), 1400(SA5), 1401(HY5), 1402(HY5), 1403(HY5), 1404(HY5), 1405(HY5), 1406(HY5), 1407(SA5), 1408(SA5), 1409(SA5), 1410(SA5), 1411(SA5), 1412(EM4), 1413(EM4), 1414(PO5), 1415(EM4)[c], 1416(PO5)[c], 1417 (HY5)[c], 1418(EM4), 1419(PO5), 1420(PO5), 1421(PO5), 1422(SA5)[c], 1423(SA5), 1424(EM4), 1425(CP2).

NOVEMBER: 1426(SM4), 1427(SM4), 1428(SM4), 1429(SM4), 1430(SM4)[c], 1431(ST6), 1432(ST6), 1433(ST6), 1434(ST6), 1435(ST6), 1436(ST6), 1437(ST6), 1438(SM4), 1439(SM4), 1440(ST6), 1441(ST6), 1442(ST6)[c], 1443(SU2), 1444(SU2), 1445(SU2), 1446(SU2), 1447(SU2), 1448(SU2)[c].

DECEMBER: 1449(HY6), 1450(HY6), 1451(HY6), 1452(HY6), 1453(HY6), 1454(HY6), 1455(HY6), 1456(HY6)[c], 1457(PO6), 1458(PO6), 1459(PO6), 1460(PO6)[c], 1461(SA6), 1462(SA6), 1463(SA6), 1464(SA6), 1465(SA6), 1466(SA6)[c], 1467(AT2), 1468(AT2), 1469(AT2), 1470(AT2), 1471(AT2), 1472(AT2), 1473(AT2), 1474(AT2), 1475(AT2), 1476(AT2), 1477(AT2), 1478(AT2), 1479(AT2), 1480(AT2), 1481(AT2), 1482(AT2), 1483(AT2), 1484(AT2), 1485(AT2)[c], 1486(BD2), 1487(BD2), 1488(PO6), 1489(PO6), 1490(BD2), 1491(BD2), 1492(HY6), 1493(BD2).

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JANUARY: 1494(EM1), 1495(EM1), 1496(EM1), 1497(IR1), 1498(IR1), 1499(IR1), 1500(IR1), 1501(IR1), 1502(IR1), 1503(IR1), 1504(IR1), 1505(IR1), 1506(IR1), 1507(IR1), 1508(ST1), 1509(ST1), 1510(ST1), 1511(ST1), 1512(ST1), 1513(WW1), 1514(WW1), 1515(WW1), 1516(WW1), 1517(WW1), 1518(WW1), 1519(ST1), 1520(EM1)[c], 1521(IR1)[c], 1522(ST1)[c], 1523(WW1)[c], 1524(HW1), 1525(HW1), 1526(HW1)[c], 1527(HW1).

c. Discussion of several papers, grouped by Divisions.